

2pm-3:40pm

EE 2400

Dr. Casey
Homework not graded

C

Noon tomorrow

when to do what with what is presented to you

CO-OP Program

6.8 μ H - use units
Micro Henrys

complex system of equations

Ti-89

$$6.8 \times 10^{-6} \text{ H} = 6.8 \text{ E-6 H} \neq 6.8 \text{ e-6 H}$$

Basic quantity: Electric Charge
unit coulomb, C.

$$q = \pm 1.602 \times 10^{-19} \text{ C}$$

like charges repel opposites attract

Voltage

Voltage is moving a charge in the presence of other charges

Voltage is path-independent

Mathematically $V_{ba} = \frac{W_{ba}}{q}$ $\frac{\text{unit of work}}{\text{unit of charge}}$

Unit analysis: $1 \text{ V} = 1 \text{ J/C}$ Joules/coulomb

Current and Magnetic Forces

current is the rate of electric charge movement

$$q \rightarrow = \leftarrow q$$

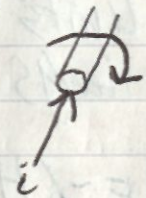
This is what we define as +ive current
current is the flow of holes

pony
Example
 $\pm$ voltage

Mathematically: $i(t) = \frac{dq(t)}{dt}$

Unit analysis: $1A = 1C/s$

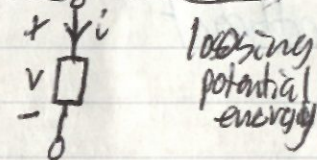
Magnetic fields: Right hand rule



Resistor Analysis I

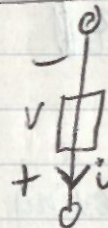
Lumped circuit elements - models for physical devices. The electric and magnetic fields are confined to the devices

passive element



losing potential energy

active element



gaining potential energy

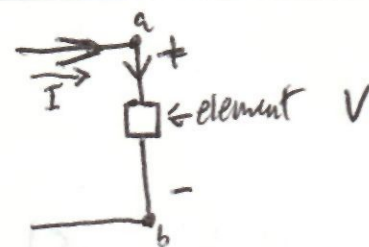
for power $p(t) = \frac{dw(t)}{dt}$
 $p = vi$ Memorize

Passive Sign Convention

$$\left[\frac{dw}{dq} \right] \left[\frac{dq}{dt} \right] = \frac{dw}{dt}$$

$$\left[1 \frac{J}{C} \right] \left[1 \frac{C}{s} \right] = 1 \frac{J}{s} = 1W$$

Watts



possible sign conventions

(direction of arrow)

If the reference direction of the current is from the + side of the voltage reference through the element to the - side of the voltage reference then $\rightarrow P = VI$ and a value $P > 0$ signifies that power is being absorbed by the element while a value of $P < 0$ signifies that power is being provided by the element.

This is called the passive sign convention.

proof

Case 1 $V > 0$ and $I > 0$

Physics: Since I is positive, charge moves from point a through the element point b. Since V is positive, charge at point a has a higher level of energy than charge at point b. The charge thus loses energy as it travels from a to b through the element, and this energy (or power) must be absorbed by the element

Tuesday July 2nd

Case 2: $V > 0$ and $I < 0$

The physics: Since I is negative, charge moves from point b through the element to point a . Since V is positive, charge at point a has a higher level of energy than charge at point b . The charge thus gains energy as it travels from b to a through the element, and this energy (or power) must be provided by the element.

Case 3: $V < 0$ and $I > 0$

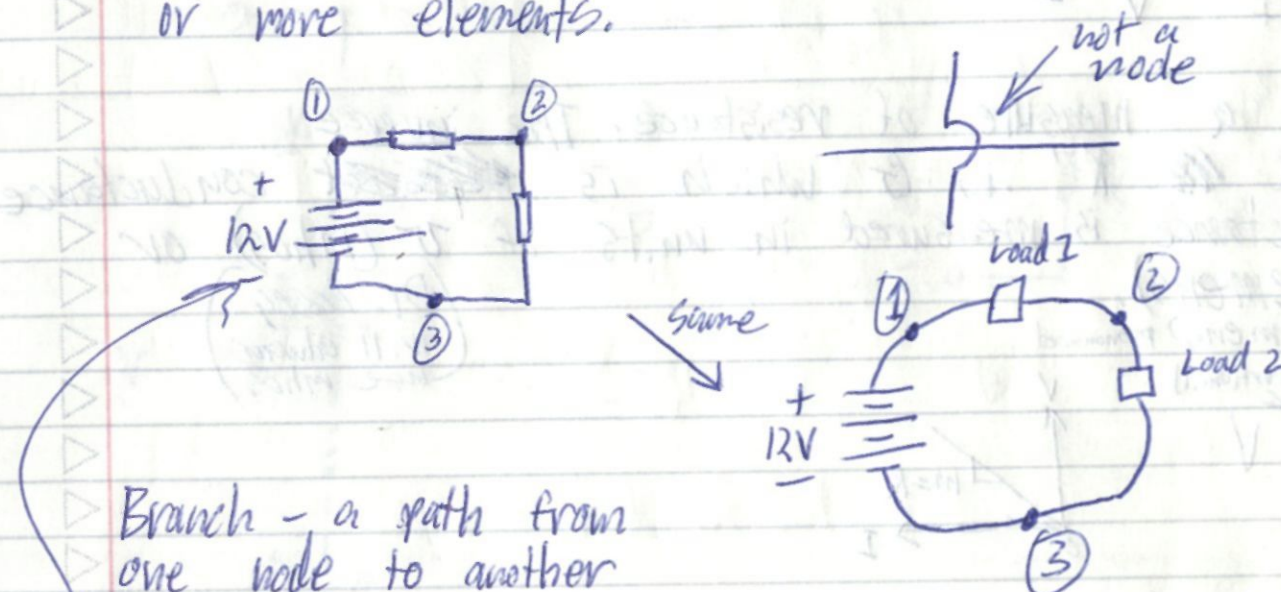
Since I is positive, charge moves from point a through the element to point b . Since V is negative, charge at point b has a ^{higher} level of energy than charge at point a . The charge thus ~~loses~~ gains energy as it travels from point a to point b (through the element), and this energy (or power) must be provided by the element.

Case 4: $V < 0$ and $I < 0$

Since I is negative charge moves from point b through the element to point a . Since V is negative charge at point a has a lower level of energy than charge at point b . The charge thus ~~loses~~ energy as it travels from

Conservation of power

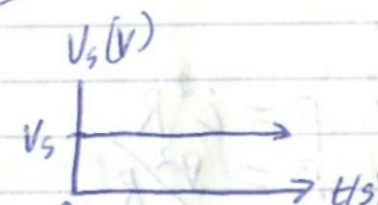
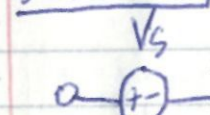
node - a point of connection between 2 or more elements.



Branch - a path from one node to another

9 branches

Sources



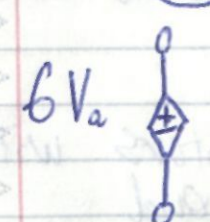
Ideal independent DC Voltage Source

Ideal independent DC Current Source

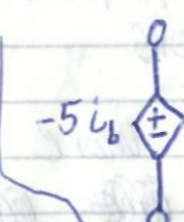


Ideal independent AC Source

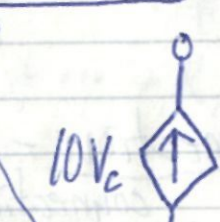
dependent on V_2



Ideal dependent Voltage controlled Voltage Source



Ideal dependent current controlled Voltage Source

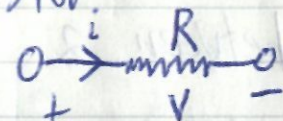


Ideal dependent voltage controlled Current source



Ideal dependent current controlled Current Source

Resistor

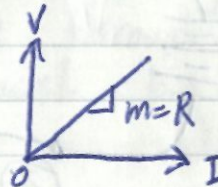


Ω
 Ω , ohms

R is a measure of resistance. The inverse of R is R^{-1} is G which is ~~conductance~~ conductance. Conductance is measured in units of Ω^{-1} (mhos) or S (Siemens).

(Ziemens) pronounced (proportionally)

$I \propto V$



(Dr. Casey will always use mhos)

* $V = IR$, Ohm's Law.

$$P = iV \rightarrow P = (iR)(i) \rightarrow P = i^2 R$$

$$i = V/R \rightarrow P = (V/R)(V) \rightarrow P = V^2/R$$

Physical implication

A series connection is when you only have a resistor connected to a node. ¹ the 2 resistors have the same current going through them

~~A parallel connection~~ ² the voltage across the 2 resistors divides proportionally across the 2 resistors

A parallel connection - 2 or more resistors with the same node connection on ~~each~~ each and

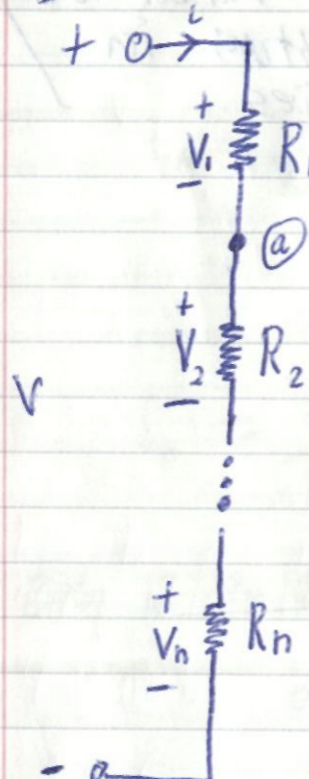
Physical implications

- 1 the voltage across the resistors is the same
- 2 The total current divides proportionally among the resistors in parallel

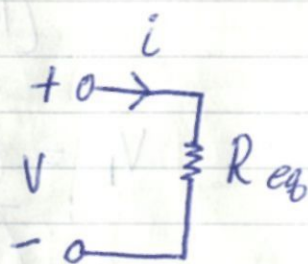
could be a test question

Proof

Resistors in series



?



$$V_1 + V_2 + \dots + V_n = V \quad V = V$$

$$iR_1 + iR_2 + \dots + iR_n = V \quad iR_{eq} = V$$

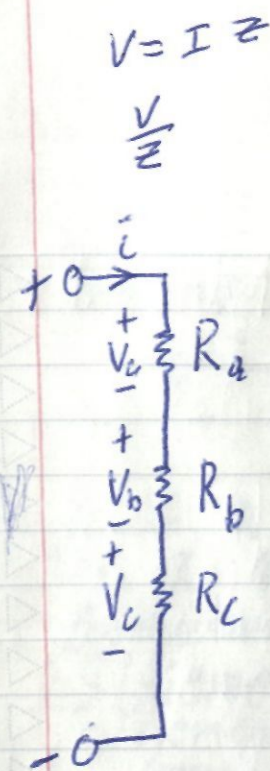
$$i(R_1 + R_2 + \dots + R_n) = V$$

$$R_1 + R_2 + \dots + R_n = V/i \quad R_{eq} = V/i$$

$$R_{eq} = R_1 + R_2 + \dots + R_n$$

QED

Quod erat demonstrandum?



Voltage Division

$$V_a = \frac{(V)(R_a)}{(R_a + R_b + R_c)}$$

(Works with any number of resistors in series)

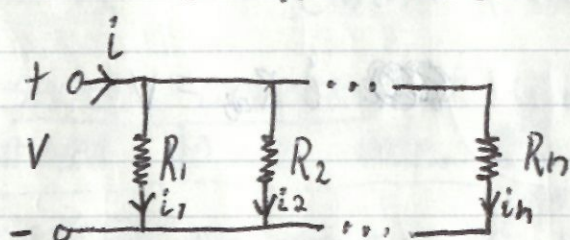
$$V_b = \frac{(V)(R_b)}{(R_a + R_b + R_c)}$$

$$V_c = \frac{(V)(R_c)}{(R_a + R_b + R_c)}$$

Wednesday, July 3

Proof

Resistors in Parallel



$$i_1 + i_2 + \dots + i_n = i$$

$$\frac{V}{R_1} + \frac{V}{R_2} + \dots + \frac{V}{R_n} = i$$

$$V \left(\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n} \right) = i$$

$$\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n} = \frac{i}{V}$$

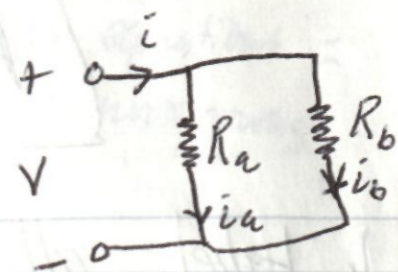
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n} \quad \text{QED}$$

$$i = \frac{V}{R_{eq}}$$

$$i = V \left(\frac{1}{R_{eq}} \right)$$

$$\frac{i}{V} = \frac{1}{R_{eq}}$$

Current Division

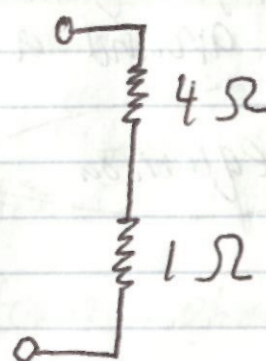


$$R_{eq} = \frac{R_a R_b}{(R_a + R_b)} \quad \text{and} \quad i \cdot R_{eq} = V$$

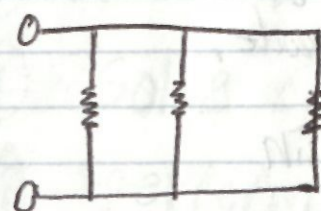
$$i_a = \frac{V}{R_a} \rightarrow i_a = \frac{i \left(\frac{R_a R_b}{(R_a + R_b)} \right)}{R_a} \rightarrow i_a = \frac{R_b \cdot i}{(R_a + R_b)}$$

$$i_b = \frac{V}{R_b} \rightarrow i_b = \frac{i \left(\frac{R_a R_b}{(R_a + R_b)} \right)}{R_b} = i_b = \frac{(R_a)(i)}{(R_a + R_b)}$$

Only 2 resistors in parallel (only works for)



$$= 5 \Omega$$



$$= 3.3 \Omega$$

Analysis by ^{circuit} Reduction

1. Label all your nodes
2. Reduce the circuit as far as necessary
3. Working backwards diagram-by-diagram exploit Ohm's law, voltage division, and current division, whenever possible, until you can calculate the requested variables.

KVL

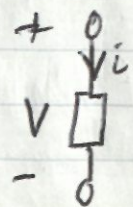
The sum of the voltage gains/rises and the voltage drops/losses around a loop equals zero

$$\sum V = 0 \quad \leftarrow \text{KVL equation}$$

Around a loop

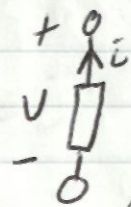
a loop is a closed path with direction that begins and ends at the same node.

Voltage drop



/ resistors drop

voltage gain



/ power supplies gain

KVL Equation

$$\sum V_{\text{losses}} = \sum V_{\text{gains}}$$

n equations = n unknowns

KVL Analysis or (loop analysis)
or (mesh analysis)
or (mesh-loop analysis)

Steps:

- ① Label all your nodes
- ② Identify and label all mesh currents
- ③ Determine which mesh currents do and do not take KVL equation and for those that do write the KVL equations
- ④ perform Ohm's Law substitutions
- ⑤ Finish building a system of equations
- ⑥ Solve the system
- ⑦ perform final calculations

mesh - a loop with no other loops inside it

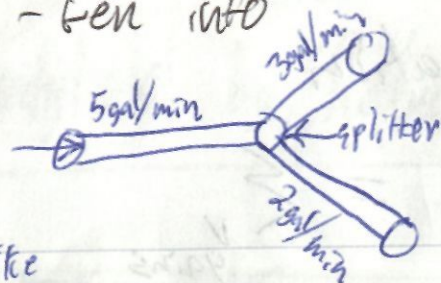
Sums of losses = Sums of gains

Exams - Gen info

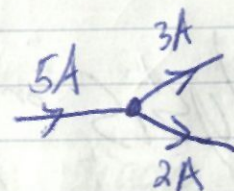
1hr and 40 min

KCL

Like hose



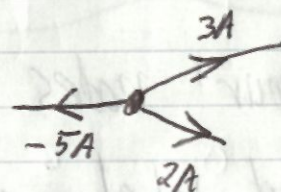
pressure - voltage
hose - wire
Water - current
splitter - node



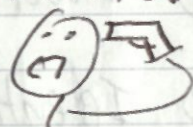
- The sum of currents leaving a node equals zero.

$$\sum i = 0A$$

Leaving
a node



$$-5A + 3A + 2A = 0$$



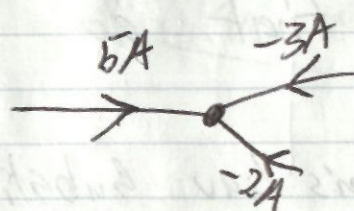
KCL equation

Equivalent Version the sum of the currents entering a node = zero

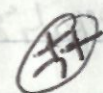
KCL equation

$$\sum i = 0A$$

Entering
a node



$$5A + -3A + -2A = 0$$



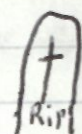
Equivalent version B: The sum of the currents entering a node equals the sum of the currents leaving a node

$$\sum i_{\text{entering a node}} = \sum i_{\text{leaving a node}}$$

$$5A = 3A + 2A$$

$$5A = 5A$$

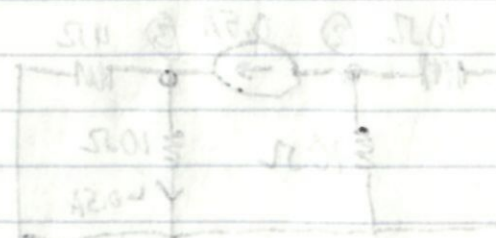
KCL equation



KCL Analysis

node analysis
or nodal analysis
or node-voltage analysis

1. label all your nodes
2. Choose and ~~label~~ label a reference node
3. finish labeling branch currents
4. determine which nodes do and do not take KCL equations and for those that do write the KCL equations
5. perform ohm's law substitutions
6. finish building a system of equations
7. Solve system (calculator)
8. final calculations



Super position

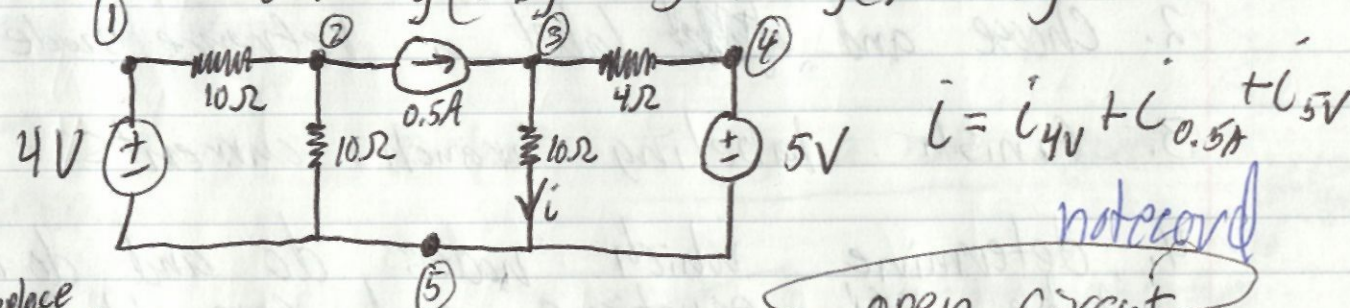
linearity - i , $g(a) = x$
 $g(2a) = 2x$

$$V = iR$$

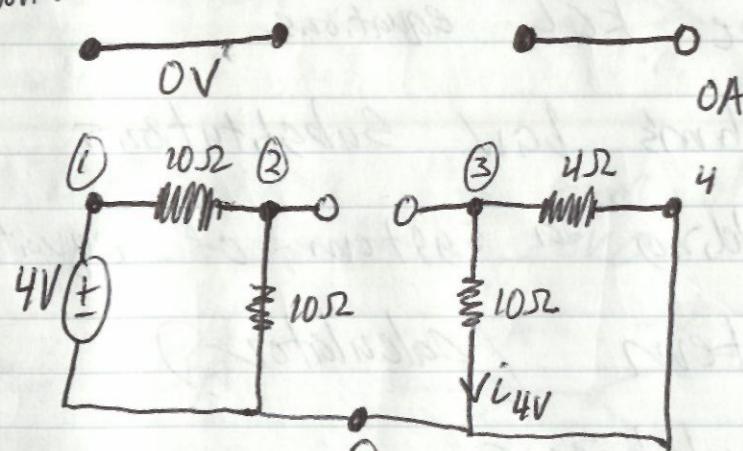
$$i = \frac{V}{R}$$

$g(b) = y$
 $g(3b) = 3y$

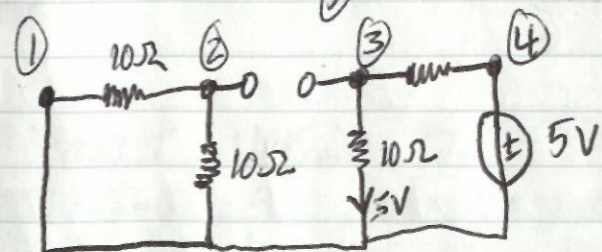
ii. $g(a+b) = g(a) + g(b) = x+y$



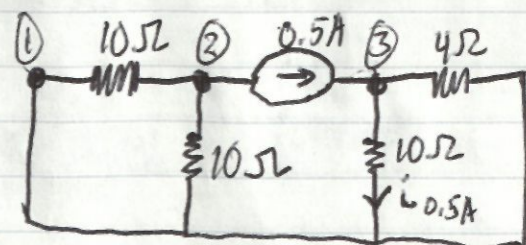
note card
 Short circuit
 replace a voltage
 always 0V but current may be non-zero



open circuit
 replace a current source with an open circuit
 always 0A but may have a non zero voltage
 $i_{4V} = 0A$, by inspection



$$i_{5V} = 0.3571A$$



$$i_{0.5A} = 0.1429A$$

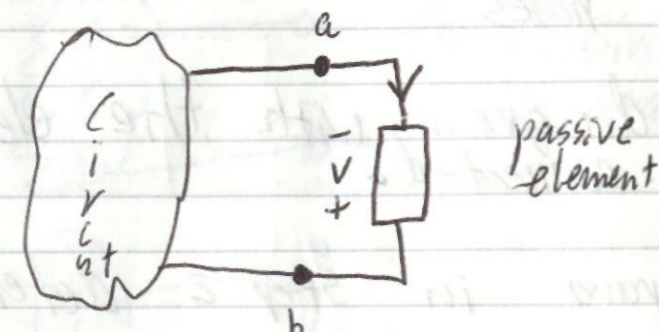
$$i = (0.1429A) + (0A) + (0.3571A)$$

$$i = 0.5$$

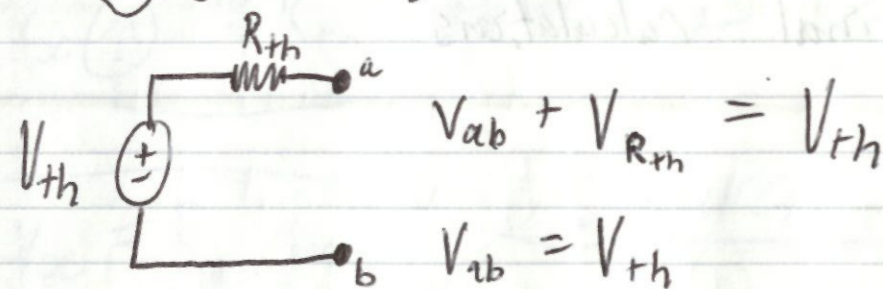
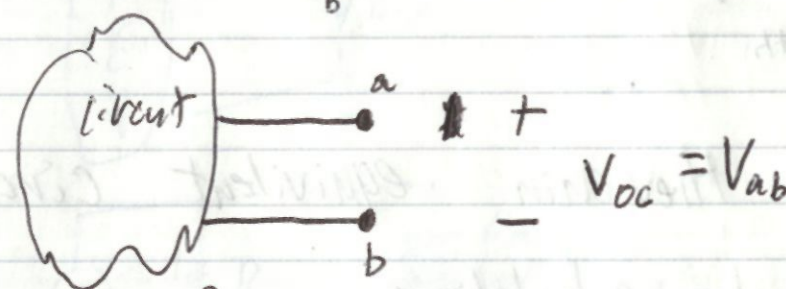
current voltage division

TEC

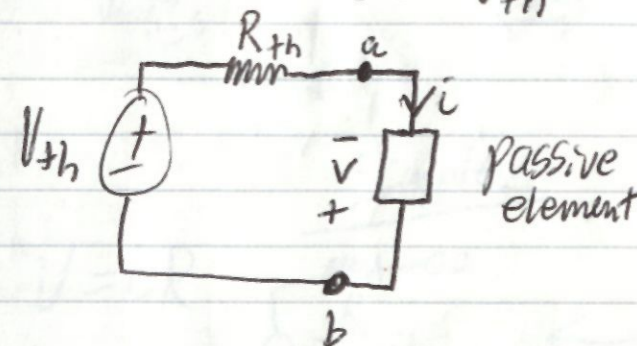
Thevenin equivalent circuit



$V_{oc} = \text{open circuit}$



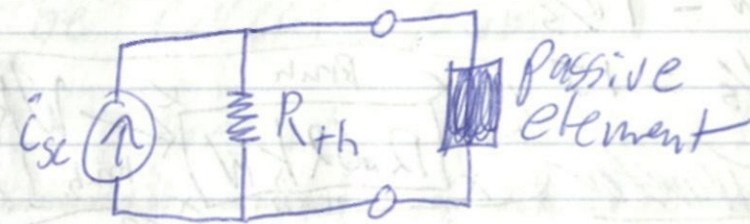
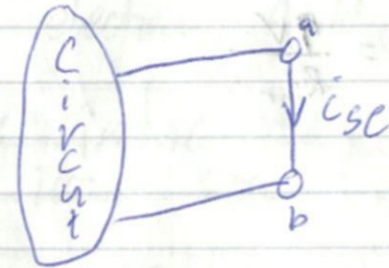
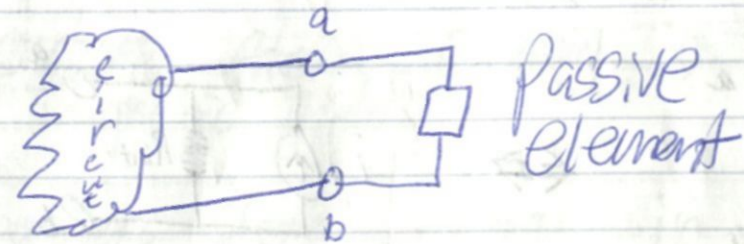
$$\therefore V_{th} = V_{oc}$$



TEC

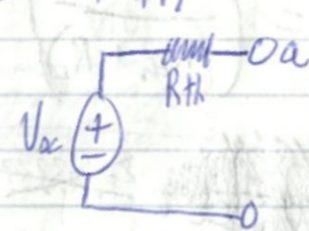
1. ~~Label~~ label all nodes
2. Redraw the diagram with the element of interest dis-connected.
3. Using the diagram in step 2 determine V_{oc} and R_{th}
 $\rightarrow$ Open circuit AKA V_{th}
4. Draw the Thevenin equivalent circuit
5. perform final calculations

Norton Equivalent Circuit

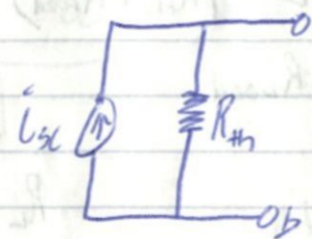


$$V_{oc} = R_{th} i_{sc}$$

Source Transformation

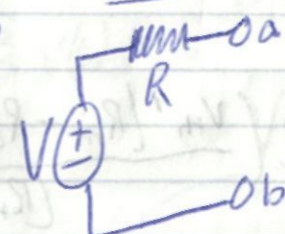


$\Leftrightarrow$

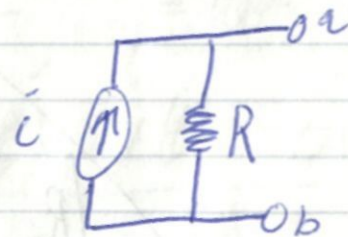


Simpler

$$V = iR$$



$\leftrightarrow$



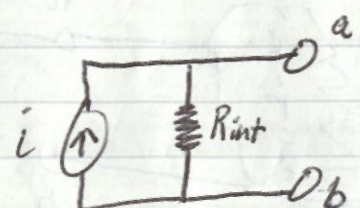
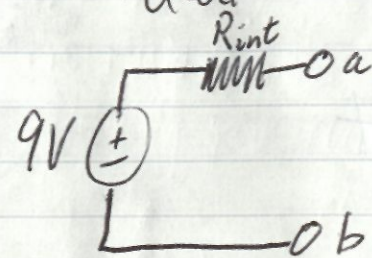
$$R_{th} = \frac{V_{oc}}{i_{sc}}$$

← use when you can't reduce
 or if there is no path with current
 or have a dependent source

$$P = i$$

R_{int} = internal Resistance

Not Ideal Sources



$$i = \frac{9V}{R_{int}}$$

Batteries: 90 mAh

$$1A = 1C/s$$

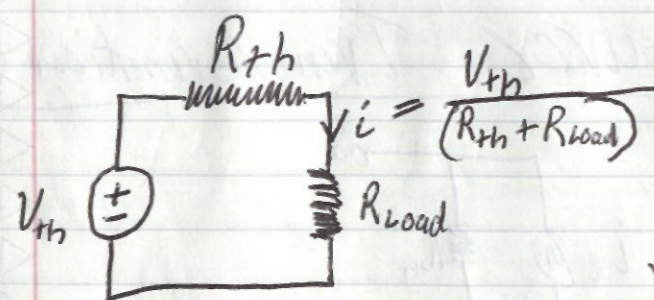
Power: 1KWh

$$1As = 1C$$

$$1W = 1J/s$$

$$1W/s = 1J$$

Maximum Power Transfer



$$P_L = \frac{V_{th}^2}{4R_L}$$

$$R_L = R_{th}$$

$$P_L(R_L) = ? = i^2 R_L$$

$$P_L(R_L) = \left(\frac{V_{th}}{R_{th} + R_L} \right)^2 R_L$$

$$R_L' 2 \left(\frac{V_{th}'(R_{th} + R_L) - V_{th}(R_{th})}{(R_{th} + R_L)^2} \right)$$

$$2 \left(\frac{V_{th}'(R_{th} + R_L) - V_{th} \cdot R_{th}}{(R_{th} + R_L)^2} \right) = 0$$

$$(R_{th} + R_L) - V_{th} = 0$$

$$(R_{th} + R_L) = V_{th}$$

$$R_L = R_{th}$$

$$R_L = V_{th} - R_{th}$$

$$V_{th}'(R_{th} + R_L) - V_{th} \cdot R_{th} = 0$$

$$V_{th}'(R_{th} + R_L) - V_{th} = 0$$

$$\frac{f'g - fg'}{g^2}$$

12.62 kW WRONG

Inductors and Capacitors

Inductors store energy in the form of a magnetic field

Capacitors store energy in the form of an electric field

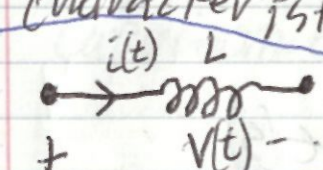
The inductor is a coil of wire, usually with an iron core

The circuit symbol is

5 characteristics to know

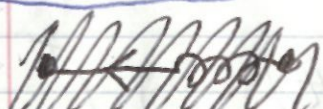
L represents the inductance in units of Henrys. [H] is the unit. The V-I characteristic is:

Know *



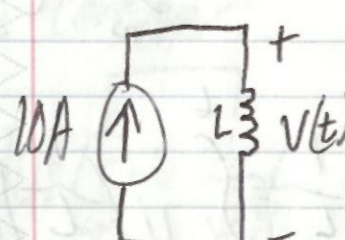
$$v(t) = L \frac{di(t)}{dt}$$

(passive element) know *



$$1V = 1$$

Consider the following



$$v(t) = L \frac{di(t)}{dt} \quad i = 10A \quad \frac{di}{dt} = 0 A/s$$

$$v(t) = L (0 A/s) \quad v(t) = 0V$$

in steady state inductors behave like a short circuit

* Know

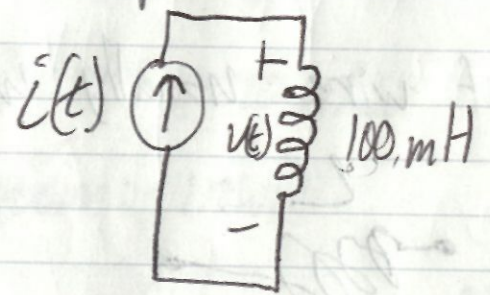
$$V(t) = L \frac{di(t)}{dt}$$

* Know

For current through an inductor to change instantaneously $\frac{di}{dt} = \infty A/s$ $V(t) = \infty V$
 ∞V is impossible. $\frac{di}{dt}$ thus the current through an inductor cannot change instantaneously

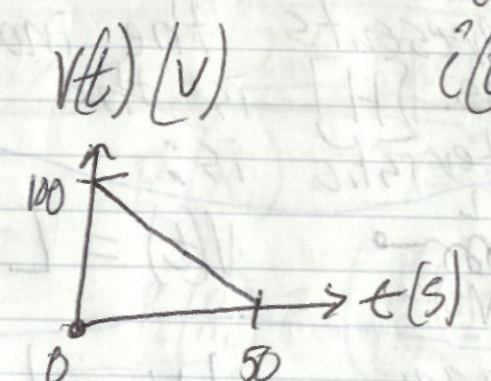
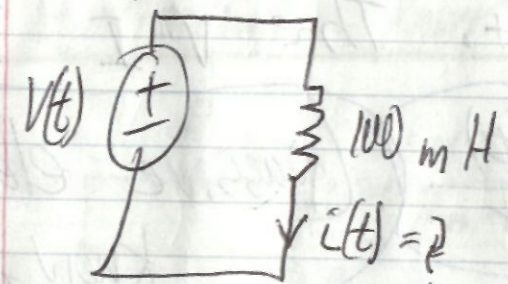
Example:

$$i(t) = \begin{cases} 0A, & t < 0 \\ 10 \sin(120\pi t)A, & t \geq 0 \end{cases}$$



$$V(t) = 120\pi \cos(120\pi t) V, t \geq 0$$

another ex:



let $i(0) = 0A$

$$V(t) = L \frac{di(t)}{dt} \Rightarrow \frac{1}{L} \int_{-\infty}^t v(\tau) d\tau = \int_{-\infty}^t \frac{di(\tau)}{d\tau} d\tau \Rightarrow \frac{1}{L} \int_{-\infty}^t v(\tau) d\tau = \int_{-\infty}^t di = i(t) - i(-\infty)$$

$$i(t) = \frac{1}{L} \int_{-\infty}^t v(\tau) d\tau \Rightarrow i(t) = \frac{1}{L} \int_0^t v(\tau) d\tau$$

$$v(t) = \begin{cases} 0V, & t < 0 \\ 100 - 2tV, & 0 \leq t \leq 50 \\ 0V, & 50 < t \end{cases}$$

$$i(t) = \frac{1}{100} \int_0^t (100 - 2\tau) d\tau = \frac{1}{100} [100\tau - \tau^2]_0^t = \frac{1}{100} (100t - t^2) = -10t^2 + 1000t A$$

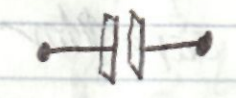
$0 \leq t \leq 50$

Integral version of V-i relation

Know

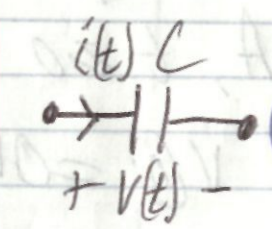
||

The capacitor is a set of charged plates separated by a gap sometimes air



The circuit symbol is: or

The v-i characteristics



Know $i(t) = C \frac{dv(t)}{dt}$

passive element

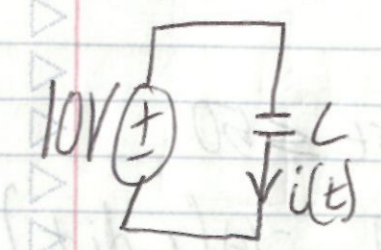
$$1A = 1FV/s$$

Capacitance is measured in farads, denoted F

ELI the ICE man

$$e(t) = \int \frac{di(t)}{dt} dt \quad i(t) = C \frac{de(t)}{dt}$$

consider



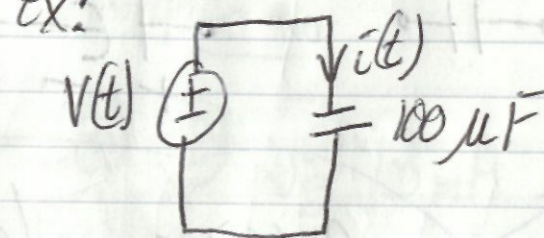
$$i(t) = C \frac{dv(t)}{dt}, \quad v(t) = 10V \rightarrow \frac{dv(t)}{dt} = 0V/s$$

$$i(t) = 0A$$

is steady state capacitors behave like an open circuit.

Know

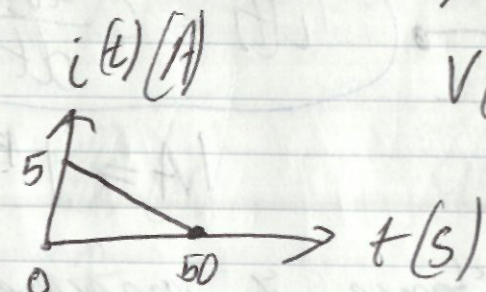
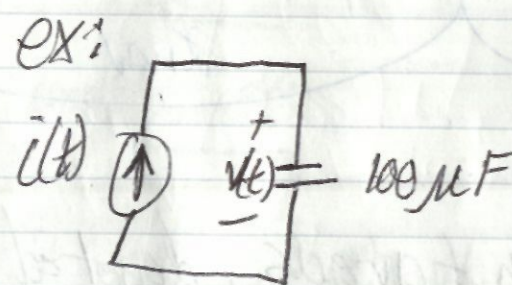
For a voltage across a capacitor to change instantaneously $\frac{dv}{dt} = \infty$ V/s $i(t) = \infty$ A. ∞ A is impossible $\rightarrow$ voltage cannot cross a capacitor cannot change instantaneously.



$$V(t) = \begin{cases} 0 \text{ V}, & t < 0 \\ 10 - 10e^{-10t} \text{ V}, & t \geq 0 \end{cases}$$

$$i(t) = 10e^{-10t} \text{ mA}, \quad t \geq 0$$

$$V(0) = 0 \text{ V}$$



$$i(t) = C \frac{dv(t)}{dt} \Rightarrow \frac{1}{C} \int_{-\infty}^t i(t) dt = \frac{dv(t)}{dt} \Rightarrow v(t) = \frac{1}{C} \int_{-\infty}^t i(t) dt$$

$$i(t) = 5 - 0.1t \text{ A}, \quad 0 \leq t \leq 50$$

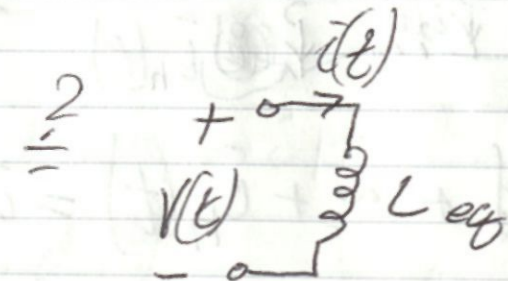
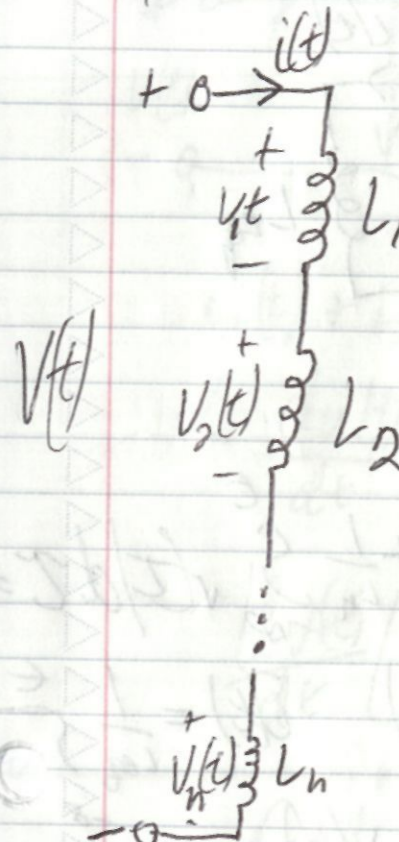
$$V(t) = -500t^2 + 50000t \text{ V}, \quad 0 \leq t \leq 50$$

Power $p(t) = v(t) \left(C \frac{dv(t)}{dt} \right)$ $p(t) = i(t) \left(L \frac{di(t)}{dt} \right)$

$$W(t) = \frac{1}{2} C v^2(t) \text{ J}$$

$$W(t) = \frac{1}{2} L i^2(t) \text{ J}$$

Inductors in Series



$$V_1(t) + V_2(t) + \dots + V_n(t) = V(t)$$

$$L_1 \frac{di(t)}{dt} + L_2 \frac{di(t)}{dt} + \dots + L_n \frac{di(t)}{dt} = V(t)$$

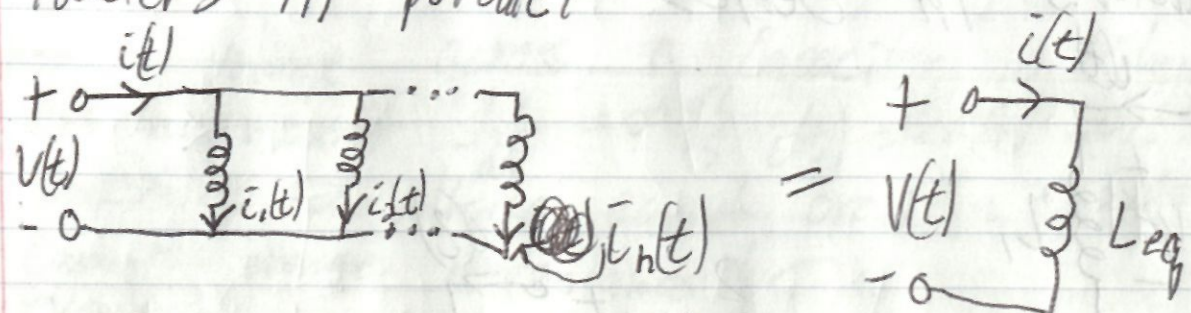
$$\frac{di}{dt} (L_1 + L_2 + \dots + L_n) = V(t)$$

$$L_1 + L_2 + \dots + L_n = \frac{V(t)}{\frac{di(t)}{dt}} \quad \frac{V(t)}{\frac{di(t)}{dt}} = L_{eq} \quad V(t) = L_{eq} \frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + \dots + L_n$$

Q.E.D

inductors in parallel



$$i_1(t) + i_2(t) + \dots + i_n(t) = i(t)$$

$$\frac{1}{L_1} \int_{-\infty}^t v(\tau) d\tau + \frac{1}{L_2} \int_{-\infty}^t v(\tau) d\tau + \dots + \frac{1}{L_n} \int_{-\infty}^t v(\tau) d\tau = i(t)$$

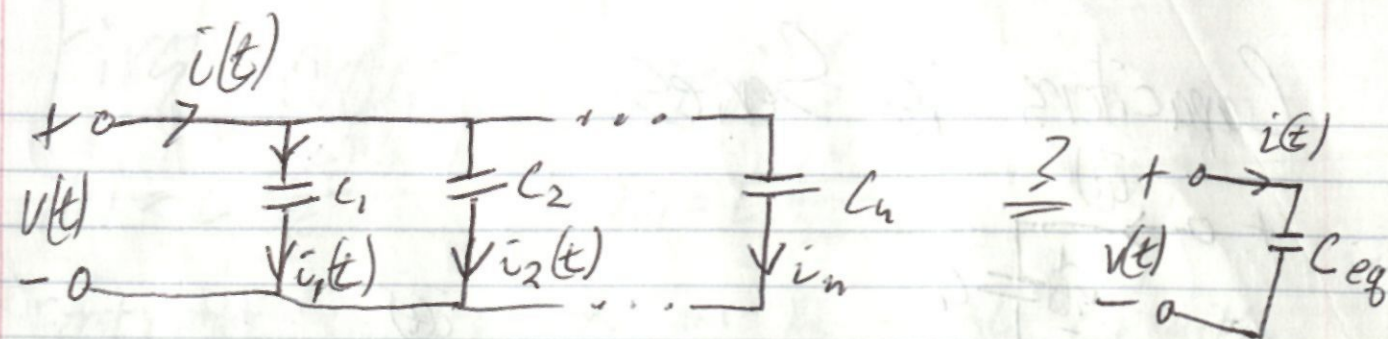
$$\int_{-\infty}^t v(\tau) d\tau \left[\frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_n} \right] = i(t) \quad i(t) = \frac{1}{L_{eq}} \int_{-\infty}^t v(\tau) d\tau$$

$$\frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_n} = \frac{i(t)}{\int_{-\infty}^t v(\tau) d\tau} = \frac{i(t)}{\int_{-\infty}^t v(\tau) d\tau} = \frac{1}{L_{eq}}$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_n}$$

QED

Capacitors in parallel



$$i_1(t) + i_2(t) + \dots + i_n(t) = i(t)$$

$$C_1 \frac{dv(t)}{dt} + C_2 \frac{dv(t)}{dt} + \dots + C_n \frac{dv(t)}{dt} = i(t)$$

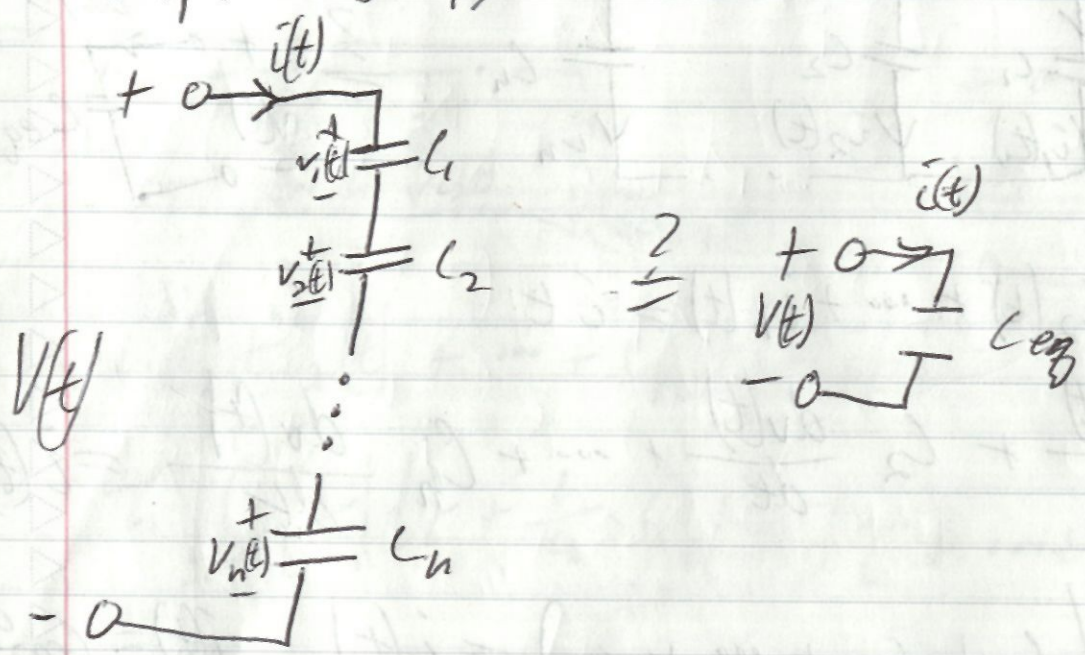
$$\frac{dv(t)}{dt} (C_1 + C_2 + \dots + C_n) = i(t) \quad i(t) = C_{eq} \frac{dv(t)}{dt}$$

$$C_1 + C_2 + \dots + C_n = \frac{i(t)}{\left(\frac{dv(t)}{dt} \right)} \quad \frac{i(t)}{\left(\frac{dv(t)}{dt} \right)} = C_{eq}$$

$$C_{eq} = C_1 + C_2 + \dots + C_n$$

QED

Capacitors in Series



$$V_1(t) + V_2(t) + \dots + V_n(t) = V(t)$$

$$\frac{1}{C_1} \int_{-\infty}^t i(\tau) d\tau \left(\frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n} \right) = V(t)$$

$$\frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n} = \frac{V(t)}{\left(\int_{-\infty}^t i(\tau) d\tau \right)} = \frac{1}{C_{eq}} \quad V(t) = \frac{1}{C_{eq}} \int_{-\infty}^t i(\tau) d\tau$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$

Q.E.D

First order Circuits

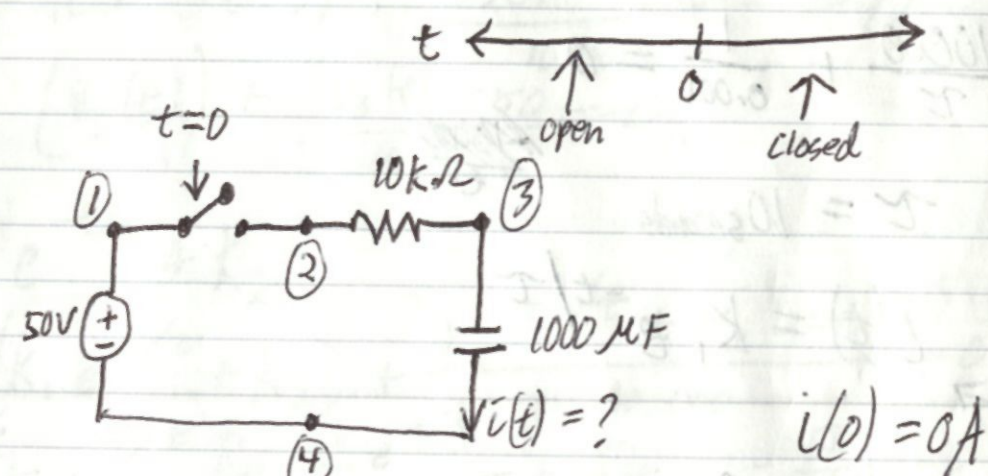
$$i(t) = C \frac{dv(t)}{dt}$$

$$V(t) = L \frac{di(t)}{dt}$$

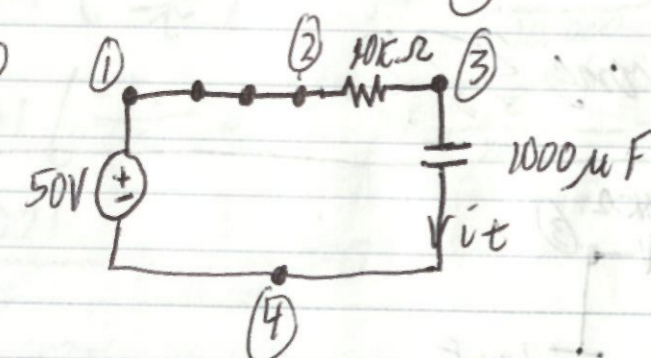
First order circuits are aka RL or RC circuits. These circuits have only one energy storage element in them - either an inductor or a capacitor.

Switch

$t < 0$



$t \geq 0$



$$V_R(t) + V_C(t) = V_S(t)$$

($t=0$)

τ - time constant

$$10,000 i(t) + \frac{1}{1E-3} \int_{-\infty}^t i(\tau) d\tau = 50$$

$$10,000 \frac{di(t)}{dt} + \frac{1}{1E-3} i(t) = 0$$

$$i(t) = K_1 e^{-t/\tau} + K_2$$

natural transient response

forced Steady state response

$$10000 \frac{d}{dt} (K_1 e^{-t/\tau} + K_2) + \frac{1}{0.001} (K_1 e^{-t/\tau} + K_2) = 0$$

$$10000 \left(-\frac{K_1}{\tau} e^{-t/\tau} \right) + \frac{1}{0.001} K_1 e^{-t/\tau} + \frac{1}{0.001} K_2 = 0$$

$$10000 \left(-\frac{K_1}{\tau} e^{-t/\tau} \right) + \frac{1}{0.001} K_1 e^{-t/\tau} = 0 \quad \frac{1}{0.001} K_2 = 0$$

$$-\frac{10000}{\tau} + \frac{1}{0.001} = 0 \quad K_2 = 0$$

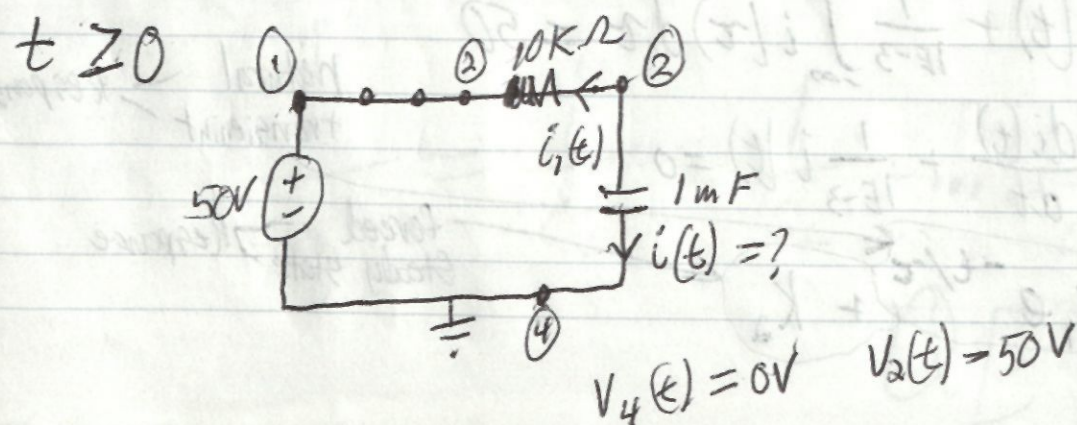
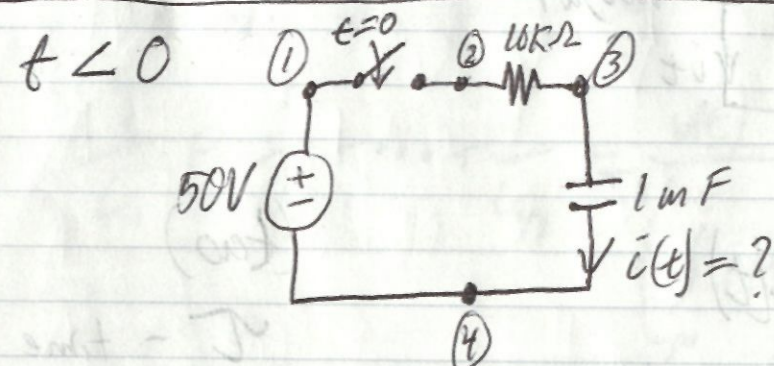
$$\tau = 10 \text{ seconds}$$

$$i(t) = K_1 e^{-t/\tau}$$

$$i(0) = 0A \quad K_1 = 0$$

$$i(t) = 0A$$

Wrong???



$$i(0) = 0A$$

$$V_3(0) = 0V$$

$$i(t) + i_1(t) = 0$$

$$0.001 \frac{d}{dt} (V_3(t) - V_4(t)) + \frac{V_3(t) - V_2(t)}{10,000} = 0$$

$$0.001 \frac{d}{dt} (V_3(t)) + \frac{V_3(t) - 50}{10,000} = 0$$

$$V_2(t) = K_1 e^{-t/\tau} + K_2$$

$$0.001 \frac{d}{dt} (K_1 e^{-t/\tau} + K_2) + \frac{K_1 e^{-t/\tau} + K_2 - 50}{10,000} = 0$$

$$0.001 \left(-\frac{K_1}{\tau} e^{-t/\tau} \right) + \frac{K_1 e^{-t/\tau}}{10,000} + \frac{K_2}{10,000} - \frac{50}{10,000} = 0$$

$$0.001 \left(-\frac{K_1}{\tau} e^{-t/\tau} \right) + \frac{K_1 e^{-t/\tau}}{10,000} = 0 \quad \frac{K_2}{10,000} - \frac{50}{10,000} = 0$$

$$-\frac{0.001}{\tau} + \frac{1}{10,000} = 0$$

$$\tau = 10 \text{ sec}$$

$$K_2 = 50$$

$$V_3(t) = K_1 e^{-t/\tau} + 50$$

$$i(0) = 0A$$

$$K_1 = -50$$

$$V_3(t) = -50 e^{-0.1t} + 50V \quad t \geq 0$$

$$V_3'(t) = 5 e^{-0.1t} \text{ V/s} \quad t \geq 0$$

$$i(t) = 5 e^{-0.1t} \text{ mA}, t \geq 0$$

Capacitor

$$i(t) = C \frac{dv(t)}{dt}$$

$$i(t) = C \frac{d}{dt} (V_3(t) - V_4(t))$$

$$i(t) = C \frac{d}{dt} \frac{dv_3(t)}{dt}$$

~~10000~~ ~~10000~~

~~10000 + 10000~~ ~~10000~~

~~10000~~ ~~10000~~

Calculus based solution - Shortcut

All elements voltages and currents will always have the same form: $K_1 e^{-t/\tau} + K_2$.
One does not have to work a calc problem but eqn.

Six Steps

1. Draw the diagram after switch action occurs.
2. Determine the initial condition.
3. Determine τ
4. Determine K_2
5. Determine K_1
6. ~~Write~~ Write the final answer.

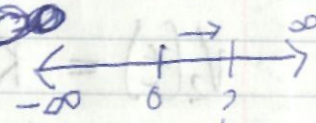
2. replace energy storage element with an empty circle

turns into

Steady state	
Capacitor	Voltage source - open circuit
inductor	Current source - closed circuit Short

The value of the source = the initial current or voltage across the element
inductor capacitor
to determine go to original diagram for initial condition

check: ① 1 or more resistor τ
② = to 0
③ how many switch actions



Determine τ

$$3. \tau = \frac{L}{R} \quad \tau = RC$$

$$4. i(t) = K_1 e^{-t/\tau} + K_2 \quad i(\infty) = K_2 = 0.2$$

$$5. K_1 = \text{step 2} - \text{step 4}$$

$$i(0) - i(\infty)$$

$$K_1 = -0.2$$

$$6. i(t) = -0.2 e^{-t/\tau} + 0.2 A, t > 0$$

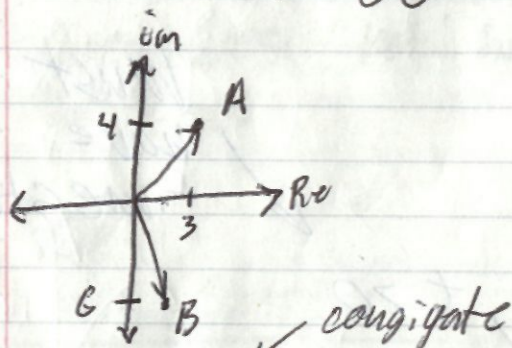
Should be at max power transfer

Must have timestamp

$$\sqrt{-1} = j = \frac{1}{-j} \quad \swarrow \text{vinculum}$$

$$A = 3 + j4 \quad \text{Re}\{A\} = 3 \quad \text{Im}\{A\} = 4$$

$$B = 2 - j6 \quad \text{Re}\{B\} = 2 \quad \text{Im}\{B\} = -6$$



$$A^* = 3 - j4 \quad \swarrow \text{conjugate}$$

$$B^* = 2 + j6$$

$$A + B = 5 - j2$$

$$A - B = 1 + j10$$

$$AB = 30 - j10$$

$$\frac{A}{B} = \frac{3 + j4}{2 - j6} \cdot \frac{2 + j6}{2 + j6} = \frac{18 + j26}{40}$$

$$\frac{6 + 18j + 8j - 24}{4 + 36}$$

$$4 + 36$$

$$0.45 + j0.85$$

$$\frac{-18 + 26j}{40} = -0.45 + j0.65$$

$$A = M \angle \phi$$

$$M = \sqrt{\text{Re}\{A\}^2 + \text{Im}\{A\}^2}$$

$$\phi = \tan^{-1} \left(\frac{\text{Im}\{A\}}{\text{Re}\{A\}} \right)$$

$$A = 5 \angle 53.13^\circ$$

$$B = 6.32 \angle -71.56^\circ$$

$$A^* = 5 \angle -53.13^\circ$$

$$B^* = 6.32 \angle 71.56^\circ$$

$$AB = 31.6 \angle -18.43^\circ$$

$$\frac{A}{B} = 0.791 \angle 124.69^\circ$$

$$A = M \angle \phi$$

$$A = M \cos(\phi) \pm j M \sin(\phi)$$

add
multiply
subtract
divide

AC Analysis

Sinusoidal Steady-State Analysis

$$v(t) = V_m \cos(\omega t + \phi) \text{ V} \leftarrow \text{unit}$$

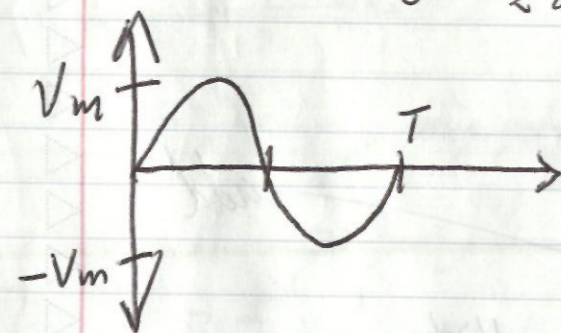
V_m → peak or amplitude value
 ω → angular frequency (rad/s)
 ϕ → phase angle
 t → time (independent variable)

$$\omega = 2\pi f$$

f = operating frequency (Hertz)

$$T = 1/f = \text{period}$$

must be referenced to cosine



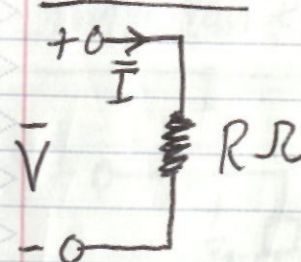
The Phasor: $\bar{V} = \frac{V_m}{\sqrt{2}} \angle \phi \text{ V}$

$\tilde{V}, \vec{V}, \vec{V}$ (with a checkmark)

← same for current except use i

Impedance $\bar{Z} \triangleq \bar{V}/\bar{I}$

Resistors



$$v(t) = V_m \cos(\omega t) \text{ V}, \quad i(t) = I_m \cos(\omega t) \text{ A}$$

$$v(t) = Ri(t)$$

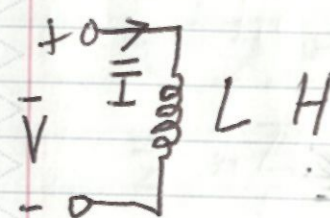
$$V_m \cos(\omega t) = R I_m \cos(\omega t)$$

$$\frac{V_m}{\sqrt{2}} \angle 0^\circ = R \left(\frac{I_m}{\sqrt{2}} \angle 0^\circ \right)$$

$$\bar{V} = R \bar{I}$$

$$\bar{V}/\bar{I} = \bar{Z}_R = R \Omega = R \angle 0^\circ \Omega$$

Inductors



$$v(t) = L \frac{di(t)}{dt}$$

$$V_m \cos(\omega t) = L \frac{d}{dt} (I_m \cos(\omega t))$$

$$V_m \cos(\omega t) = -L (\omega I_m \sin(\omega t))$$

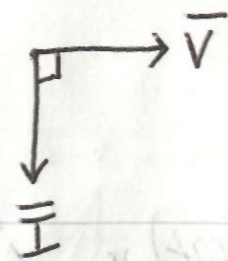
$$V_m \cos(\omega t) = -\omega L I_m \cos(\omega t - 90^\circ)$$

$$\frac{V_m}{\sqrt{2}} \angle 0^\circ = -\omega L \left(\frac{I_m}{\sqrt{2}} \angle -90^\circ \right)$$

$$\bar{V} = -\omega L \left(\frac{I_m}{\sqrt{2}} \angle 0^\circ \right) (1 \angle -90^\circ)$$

$$\bar{V} = -\omega L (\bar{I}) (1 \angle -90^\circ)$$

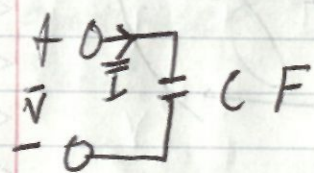
$$\bar{V}/\bar{I} = \bar{Z}_L = \omega L \angle 90^\circ \Omega = j\omega L \Omega$$



Lags

$\bar{V}$ lags behind $\bar{I}$ by 90°
Lags

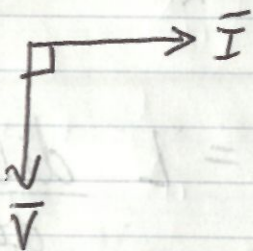
Capacitor



$$i(t) = C \frac{dv(t)}{dt}$$

$$\bar{V}/\bar{I} = \bar{Z}_C = \frac{1}{j\omega C} \Omega \equiv \frac{-j}{\omega C} \Omega$$

$$\equiv \frac{1}{\omega C} \angle -90^\circ \Omega$$



~~XXXXXXXXXXXX~~

$\bar{V}$ Leads $\bar{I}$ by 90°

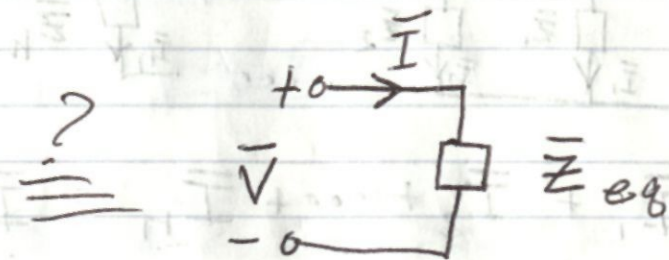
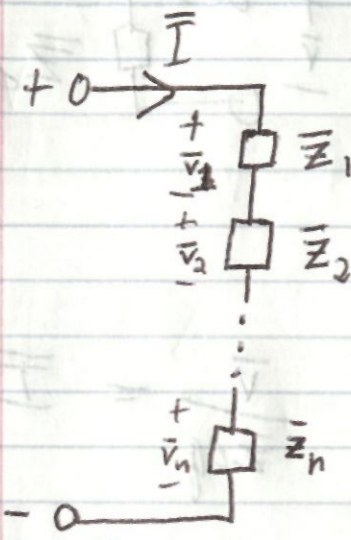
$$\bar{Z} = R + jX \Omega$$

Resistance Ω Reactance

$$\bar{Z}^{-1} = \bar{Y} = G + jB$$

admittance conductance Ω susceptance

impedances in Series



$$\bar{V}_1 + \bar{V}_2 + \dots + \bar{V}_n = \bar{V}$$

$$\bar{I} \bar{Z}_1 + \bar{I} \bar{Z}_2 + \dots + \bar{I} \bar{Z}_n = \bar{V}$$

$$\bar{I} \bar{Z}_{eq} = \bar{V}$$

$$\bar{I} (\bar{Z}_1 + \bar{Z}_2 + \dots + \bar{Z}_n) = \bar{V}$$

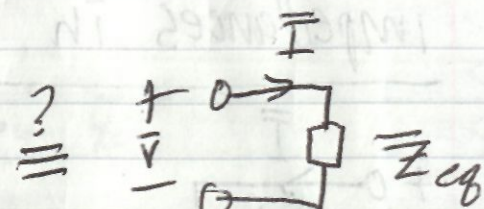
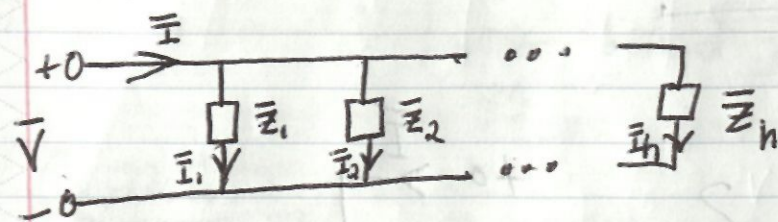
$$\bar{Z}_1 + \bar{Z}_2 + \dots + \bar{Z}_n = \bar{V}/\bar{I}$$

$$\bar{Z}_{eq} = \bar{V}/\bar{I}$$

$$\bar{Z}_{eq} = \bar{Z}_1 + \bar{Z}_2 + \dots + \bar{Z}_n$$

QED

impedances in parallel



$$\bar{I}_1 + \bar{I}_2 + \dots + \bar{I}_n = \bar{I}$$

$$\frac{\bar{V}}{\bar{Z}_1} + \dots = \bar{I}$$

$$\frac{\bar{V}}{\bar{Z}_{eq}} = \bar{I}$$

$$\bar{V} \left(\frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2} + \dots + \frac{1}{\bar{Z}_n} \right) = \bar{I}$$

$$\frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2} + \dots + \frac{1}{\bar{Z}_n} = \frac{\bar{I}}{\bar{V}} \quad \frac{1}{\bar{Z}_{eq}} = \frac{\bar{I}}{\bar{V}}$$

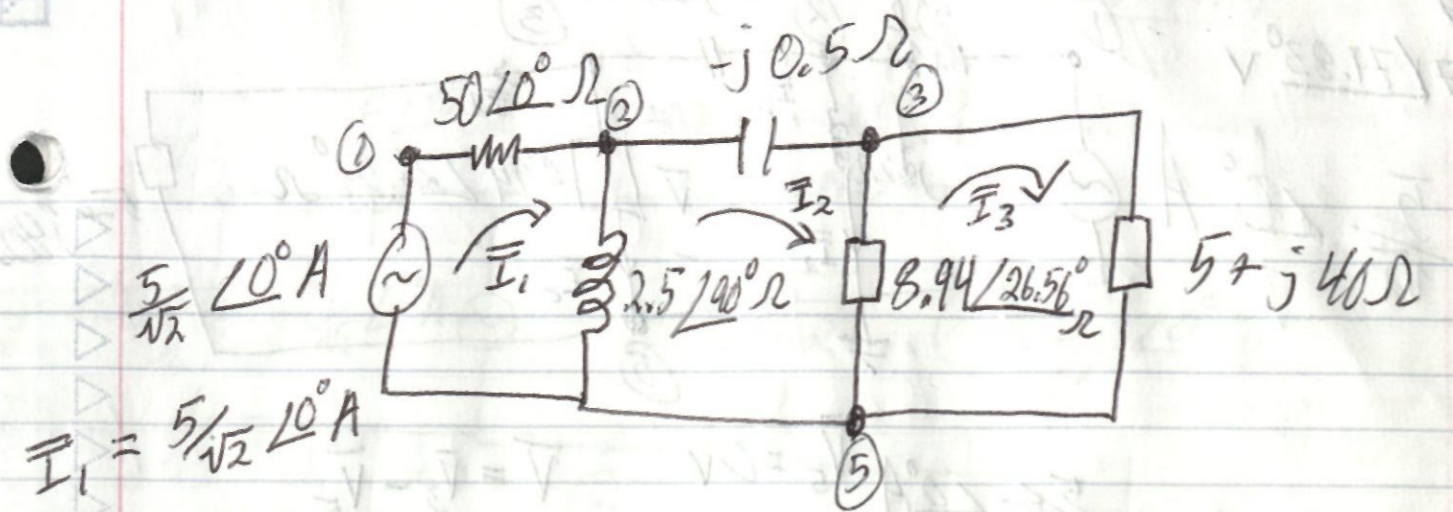
$$\frac{1}{\bar{Z}_{eq}} = \frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2} + \dots + \frac{1}{\bar{Z}_n}$$

QED

AC Analysis (steps)

1. Label all your nodes
2. Note the value of ω (omega) (rad/s)
3. Transform the diagram to the frequency domain
4. Unless otherwise noted, pick a analysis technique and analyze the circuit

to find Polar value of ~~voltage~~ ^{AC} source ~~amplitude~~ ^{divide} by $\sqrt{2}$ and ~~the~~ the angle is the phase in the Cos.
 for a capacitor $\rightarrow \frac{-j}{\omega C} \Omega$
 for an inductor $\rightarrow j\omega L \Omega$

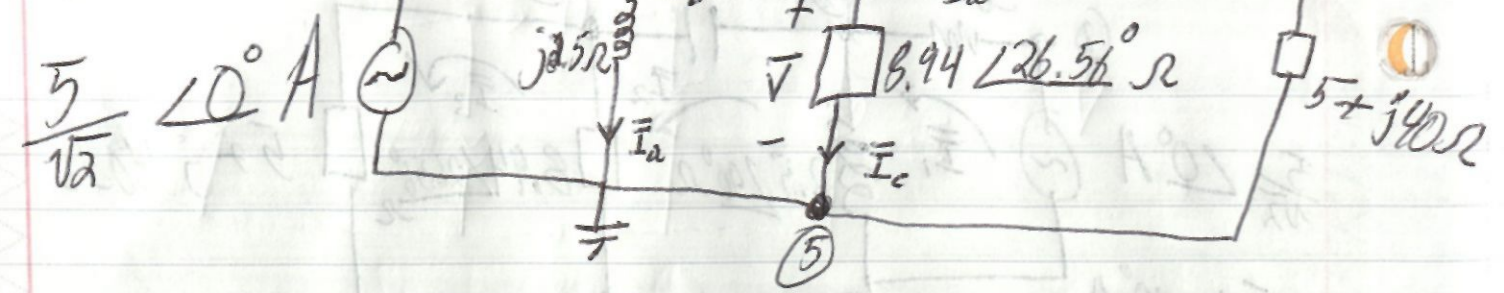


$$\bar{I}_1 = \frac{5}{\sqrt{2}} \angle 0^\circ \text{ A}$$

$$\begin{aligned} \textcircled{2} & (2.5 \angle 90^\circ - j0.5 + 8.94 \angle 26.56^\circ) \bar{I}_2 - 8.94 \angle 26.56^\circ \bar{I}_3 \\ \textcircled{3} & -(8.94 \angle 26.56^\circ) \bar{I}_2 + (8.94 \angle 26.56^\circ + 5 + j40) \bar{I}_3 = 0 \\ & = \left(\frac{5}{\sqrt{2}} \angle 0^\circ \right) (2.5 \angle 90^\circ) \end{aligned}$$

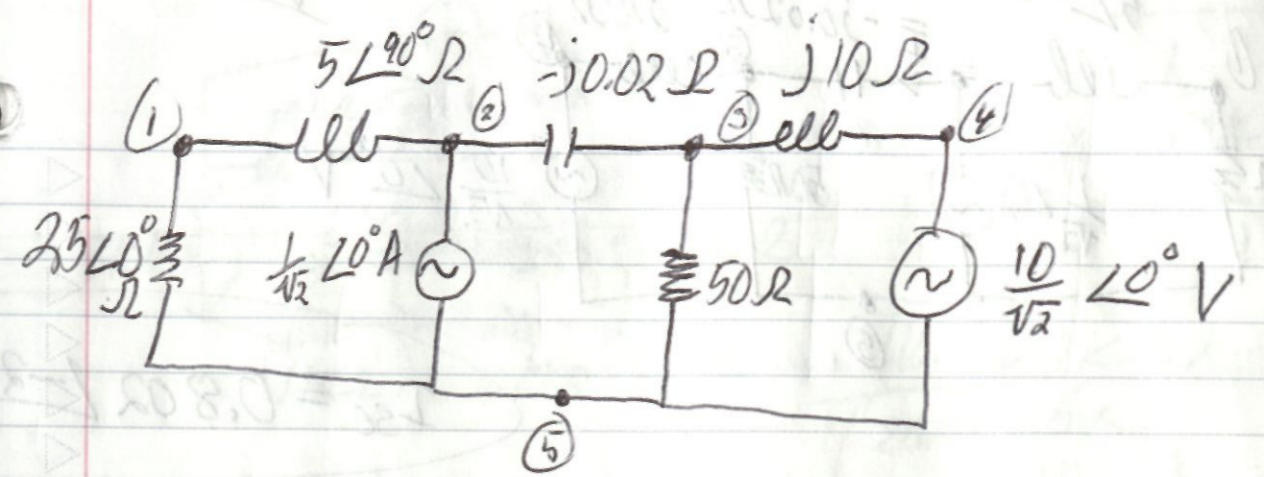
$$\bar{V} = (8.94 \angle 26.56^\circ \Omega) (\bar{I}_2 - \bar{I}_3)$$

7.57 $\angle 79.83^\circ$ V

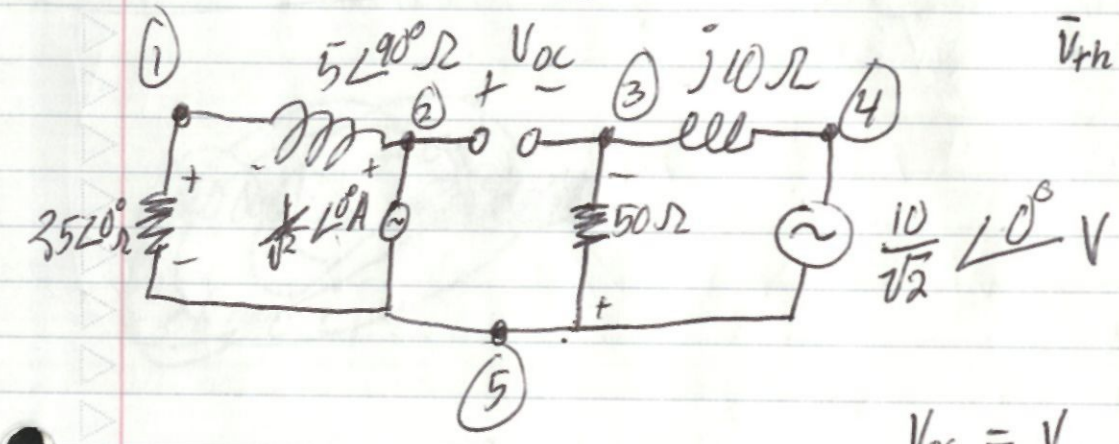


$\bar{V}_5 = 0V$
 $\bar{V} = \bar{V}_3 - \bar{V}_5$
 (2) $\frac{\bar{V}_1 - \bar{V}_2}{50} = \frac{\bar{V}_2 - \bar{V}_3}{-j0.5} + \frac{\bar{V}_2}{j2.5}$
 (3) $\frac{\bar{V}_2 - \bar{V}_3}{-j0.5} = \frac{\bar{V}_3}{5 + j40} + \frac{\bar{V}_3}{8.94 \angle 26.56^\circ}$

Thevenin

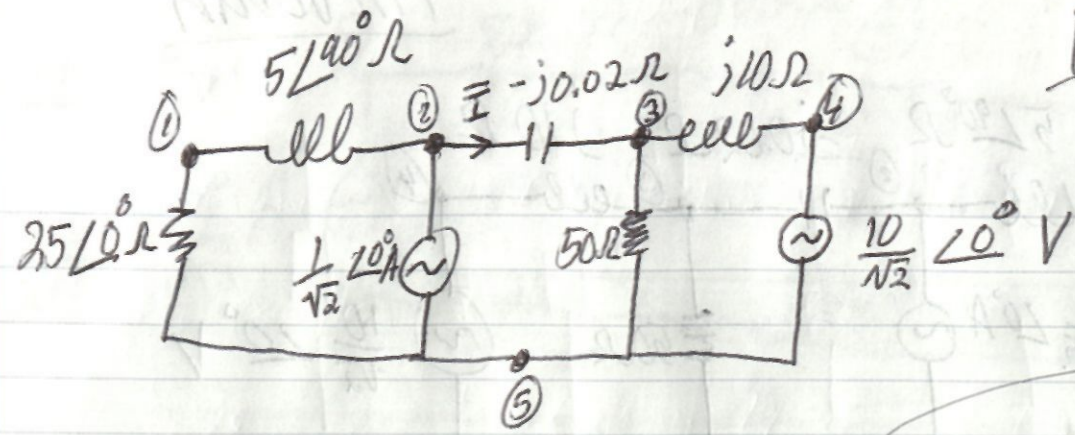


$\bar{V}_{th} = 24.57 \angle 5.06^\circ$ V



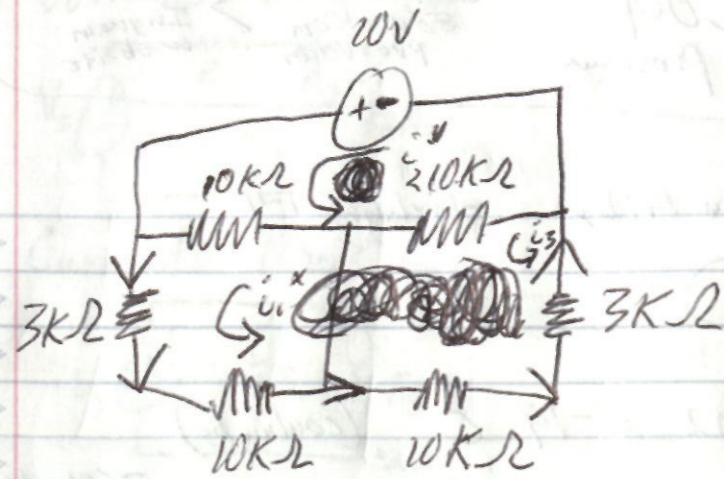
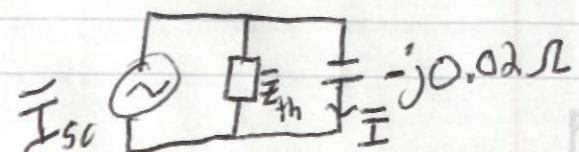
$V_{oc} = V_{21} + V_{15} + V_{53}$

$\bar{Z}_{th} = 30.63 \angle 28.49^\circ \Omega$



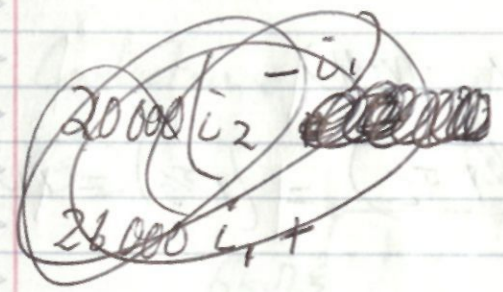
Norton

$$I_{sc} = 0.802 \angle -23.41^\circ \text{ A}$$



$$\frac{1}{26k\Omega} + \frac{1}{20k\Omega}$$

$$\frac{(10)(20k\Omega)}{(20k\Omega + 26k\Omega)}$$



$$20000(i_2 - i_1) = 10$$

$$26000i_1 + 20000(i_1 - i_2) = 0$$

Circuits II

EE 32100

Coop
Program

Cooperative
Education
Program

on
Ingram
website

* Read Announcements - HW1 due Friday - Prelabs 1-4

Blank examples (Printout)

Charge $q = \pm 1.602 \times 10^{-19} \text{ C}$ (coulombs)

Current $i(t) = \frac{dq(t)}{dt} \text{ A}$ (amps) $\nearrow \text{Coulombs/sec}$

Potential

voltage $1 \text{ V} = 1 \text{ V}$ (volt)

Power $p(t) = i(t) v(t) = (1 \text{ A})(1 \text{ V}) = 1 \text{ W}$ (watt)

Ohm's Law $v(t) = R \cdot i(t)$

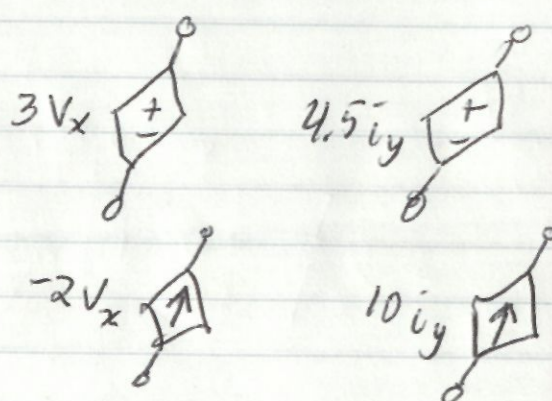
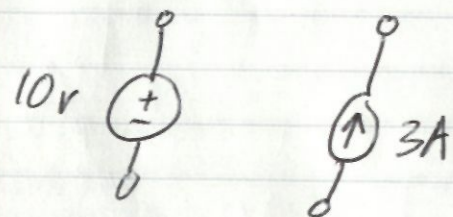
Node - a point of connection between two or more elements

Branch - a path from one node to another

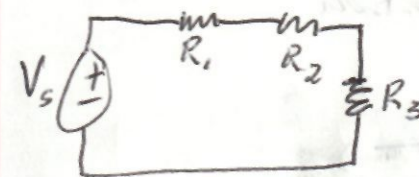
Loop - a path which begins and ends at the same node

Mesh - a loop with no other loops inside it

Ideal independent sources Dependent sources

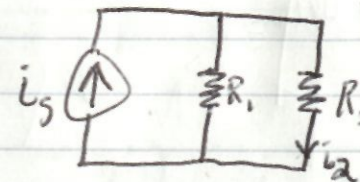


Voltage Division



$$V_{R_3} = \frac{V_s R_3}{R_1 + R_2 + R_3}$$

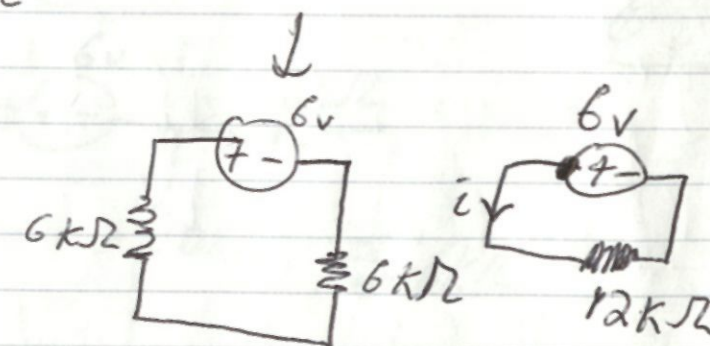
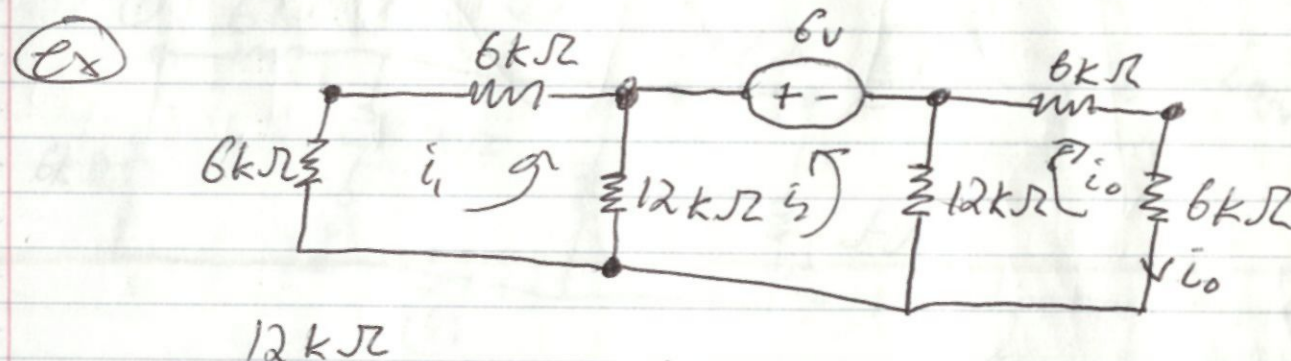
Current Division



$$i_2 = \frac{i_s R_1}{R_1 + R_2}$$

$$P = iV = i^2 R = \frac{V^2}{R}$$

Conductance $G = R^{-1}$ or S uhos Siemens



$$P = Vi$$

$$V = iR$$

$$i = 0.5 \text{ A}$$

$$i_0 = \frac{0.5 \cdot 12}{24} = 0.25$$

$$i_1 6k\Omega + i_1 6k\Omega$$

$$i_1(6k\Omega) + i_1(6k\Omega) + i_2(2k\Omega) = 0$$

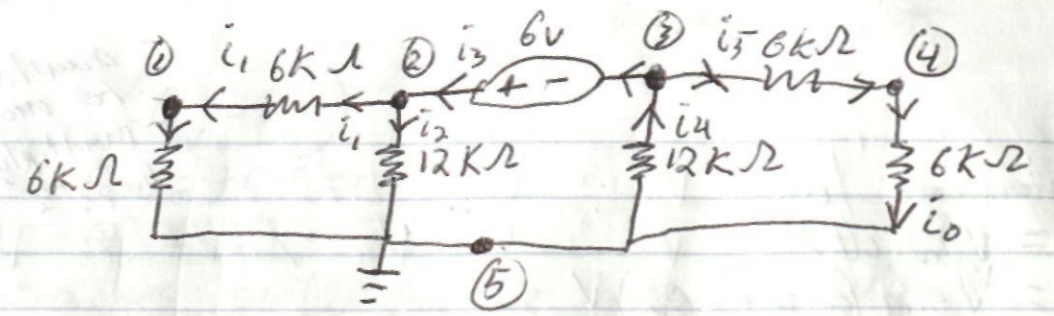
$$(12k\Omega)i_2 + (12k\Omega)i_2 = 6$$

$$i_0(12k\Omega) + i_0(6k\Omega) + i_0(6k\Omega) = 0$$

$$\frac{(-6V)(6k\Omega)}{12k\Omega} = 3V$$

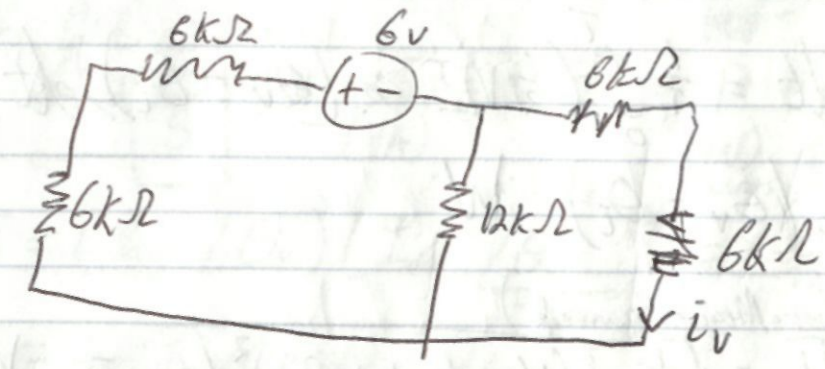
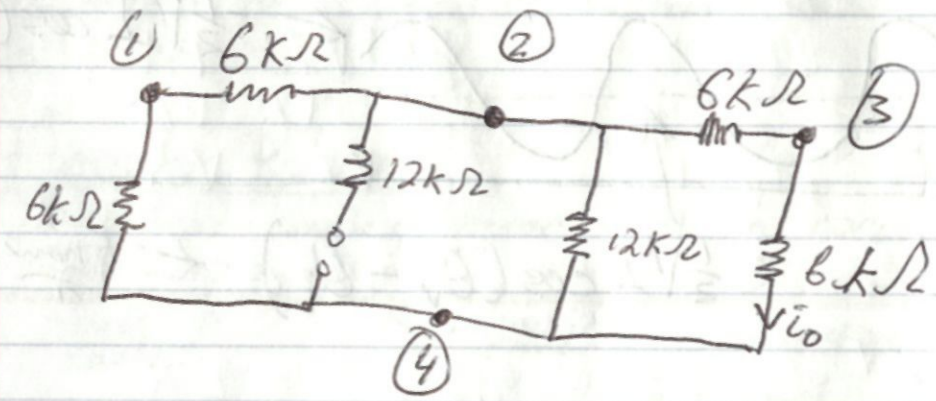
$$\frac{-3V}{12k\Omega}$$

Superposition
KVL
KCL
Nodal
Reduction

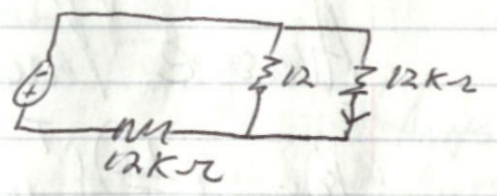


$$(2) \quad i_3 = i_1 + i_2$$

$$(3) \quad i_4 = i_5 + i_3$$



$$i_0 = i_{0_{6V}} + i_{0_{0.5A}}$$



Power

Instantaneous Power

$$p(t) = v(t) i(t)$$

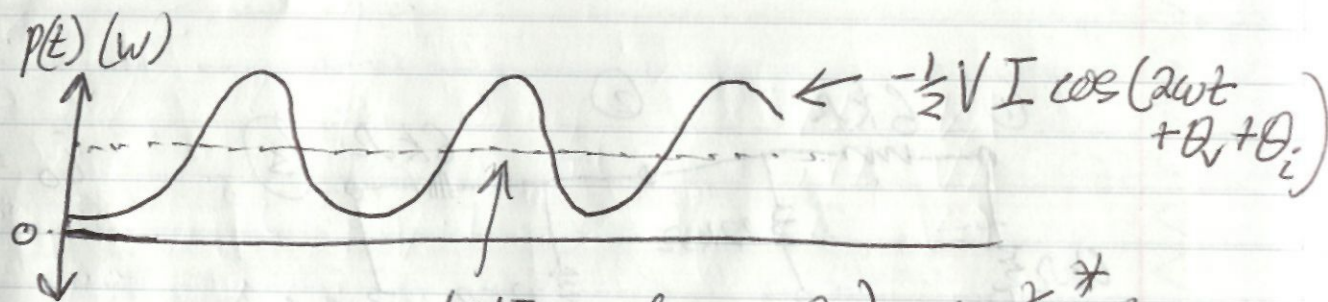
$$v(t) = V \sin(\omega t + \theta_v) \text{ V}$$

$$i(t) = I \sin(\omega t + \theta_i) \text{ A}$$

$$p(t) = [V \sin(\omega t + \theta_v) \text{ V}] [I \sin(\omega t + \theta_i) \text{ A}]$$

$$p(t) = VI \left\{ \frac{1}{2} \cos[(\omega t + \theta_v) - (\omega t + \theta_i)] - \frac{1}{2} \cos[(\omega t + \theta_v) + (\omega t + \theta_i)] \right\}$$

$$p(t) = \frac{1}{2} VI \cos(\theta_v - \theta_i) - \frac{1}{2} VI \cos(2\omega t + \theta_v + \theta_i)$$



Average Power

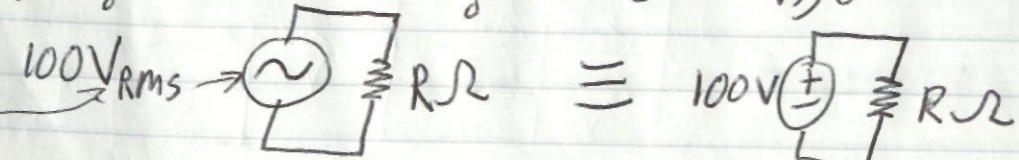
$$P_{avg} = \left(\frac{1}{T} \right) \int_0^T p(t) dt = \frac{1}{T} \int_0^T \left(\frac{1}{2} VI \cos(\theta_v - \theta_i) \right) dt$$

$$P_{avg} = \frac{1}{2} VI \cos(\theta_v - \theta_i) \text{ W}$$

RMS Values (Root-Mean-Square)

$$V_{RMS} = \sqrt{\frac{1}{T} \int_0^T v^2(t) dt} = \sqrt{\frac{1}{T} \int_0^T (V \sin(\omega t + \theta_v))^2 dt} = \frac{V}{\sqrt{2}} \text{ V}$$

Use either
on Variable
or on unit
or both



amplitude
to magnitude
multiply
by $\frac{1}{\sqrt{2}}$

power factor (no units)

* Know
memorize *

$$pf = \cos(\theta_v - \theta_i)$$

$$\text{if } \theta_v - \theta_i = \pm 90^\circ \Rightarrow pf = 0$$

$$\text{if } \theta_v - \theta_i = 0^\circ \Rightarrow pf = 1$$

One attaches two(2) possible terms to pf

$$\theta_v - \theta_i > 0 \rightarrow \underline{I} \text{ lags } \underline{V} \rightarrow \text{lagging}$$

$$\theta_v - \theta_i < 0 \rightarrow \underline{I} \text{ leads } \underline{V} \rightarrow \text{leading}$$

$$\theta_v - \theta_i = 0 \rightarrow \underline{I} \text{ and } \underline{V} \text{ are in phase (no term)}$$

$$pf = 1 \rightarrow \text{resistive element load (unity pf)}$$

$$pf \text{ lag} \rightarrow \text{inductive element load}$$

$$pf \text{ lead} \rightarrow \text{capacitive element load}$$

$$\text{ex) } pf = 0.76, \text{ lag}$$

Complex Power

we define complex power with a circuit element as $\bar{S}$.

$$\bar{S} = P + jQ$$

real power $\uparrow$ reactive power

$\bar{S}$ is a phasor quantity
reactive power does exist

By Definition...



$$\bar{S} \triangleq \underline{V} \underline{I}^* = P + jQ$$

(VA) (W) (VARs)

coke

(VA) Volt-Amps
(VAR) Volt-Amps
reactive

$$\bar{S} = \left(\frac{V}{\sqrt{2}} \angle \theta_v \right) \left(\frac{I}{\sqrt{2}} \angle \theta_i \right)^* = \left(\frac{V}{\sqrt{2}} \angle \theta_v \right) \left(\frac{I}{\sqrt{2}} \angle -\theta_i \right)$$

$$= \frac{1}{2} VI \angle \theta_v - \theta_i \text{ VA}$$

$$\underbrace{\frac{1}{2} VI \cos(\theta_v - \theta_i)}_P + j \underbrace{\left(\frac{1}{2} VI \sin(\theta_v - \theta_i) \right)}_Q$$

$$\bar{S} = P + jQ$$

Consider an impedance ...

$$\bar{V} = V \angle \theta_v \quad \bar{I} = I \angle \theta_i \quad \bar{Z} = Z \angle \theta_z$$

$$\bar{S} = \bar{V} \bar{I}^* = \frac{1}{2} VI \angle \theta_v - \theta_i \text{ VA}$$

but $\bar{V} = \bar{I} \bar{Z} \Rightarrow \bar{S} = |\bar{I}|^2 \bar{Z} \text{ VA}$

also $\bar{I} = \frac{\bar{V}}{\bar{Z}} \Rightarrow \bar{S} = \bar{V} \left(\frac{\bar{V}}{\bar{Z}} \right)^* \Rightarrow \bar{S} = \frac{|\bar{V}|^2}{\bar{Z}^*} \text{ VA}$

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{V}{I} \angle \theta_v - \theta_i$$

$$\bar{Z}_L = Z \angle 90^\circ \Omega \quad \text{pf} = 0, \text{ lag} \quad \bar{S} = jVI \text{ VA}$$

$$\bar{Z}_C = Z \angle -90^\circ \Omega \quad \text{pf} = 0, \text{ lead} \quad \bar{S} = -jVI \text{ VA}$$

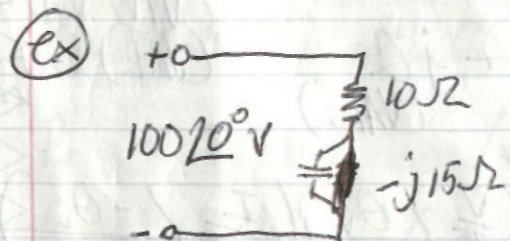
$$\bar{Z}_R = Z \angle 0^\circ \Omega \quad \text{pf} = 1 \quad \bar{S} = VI \text{ VA}$$

Example $\bar{V} = 100 \angle 0^\circ \text{ V}$

$$\bar{I} = \frac{100 \angle 0^\circ}{10 + j15} = 5.55 \angle -56.3^\circ \text{ A}$$

$$P = 307.69 \text{ W} = I^2 R$$

$$Q = 461.538 \text{ VARs} = I^2 X$$



$$\bar{I} = 5.55 \angle -56.3^\circ \text{ A}$$

$$P = 308 \text{ W}$$

$$Q = -462 \text{ VARs}$$

Element	absorbs		supplies	
	+Q	-Q	+Q	-Q
capacitor		✓	✓	
inductor	✓			✓

← for Reactive Power

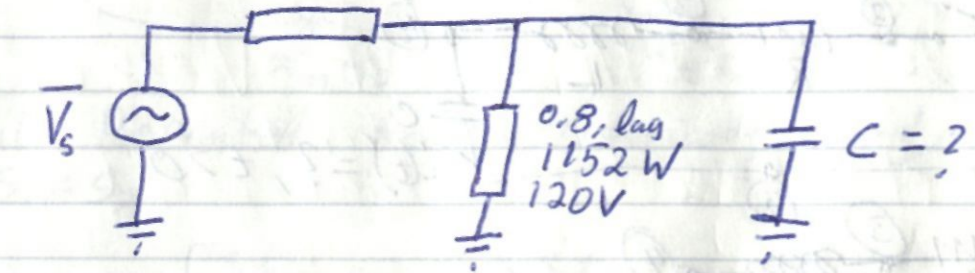
(Power factor correction)

$$\tan \theta = \frac{Q}{P}$$

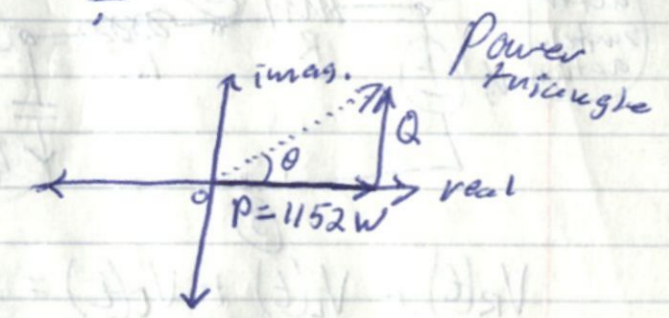
Correct the pf to 0.95, lag.

$$\text{pf} = \cos(\theta_v - \theta_i)$$

$$\bar{Z}_{int} = 1 + j2 \Omega$$



$$X_c = \frac{|\bar{V}|^2}{Q_{cap}} = \frac{1}{\omega C}$$



Correcting to 0.95

$$\cos^{-1}(0.95) = 18.19^\circ = \alpha$$



$$Q_{new} = 378 \text{ VARs}$$

$$Q_{cap} = Q - Q_{new}$$

$$Q_{cap} = 486$$

$$\frac{120^2}{486} = \frac{1}{\omega C}$$

$$29.63 = \frac{1}{\omega C}$$

$$0.03375 \text{ F}$$

$$\theta = \cos^{-1}(0.8) \quad \tan \theta = \frac{Q}{P}$$

$$Q = 864 \text{ VARs}$$

(the hypotenuse) $|\bar{S}| = 1440 \text{ VA}$
Apparent Power

$$C = \frac{Q_{cap}}{|\bar{V}|^2 \omega}$$

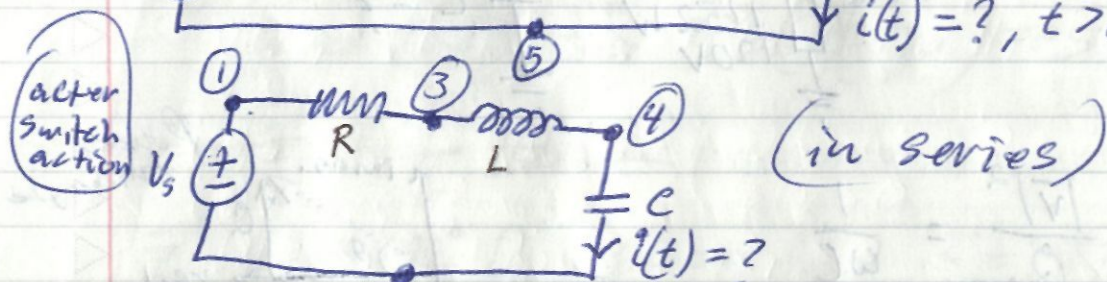
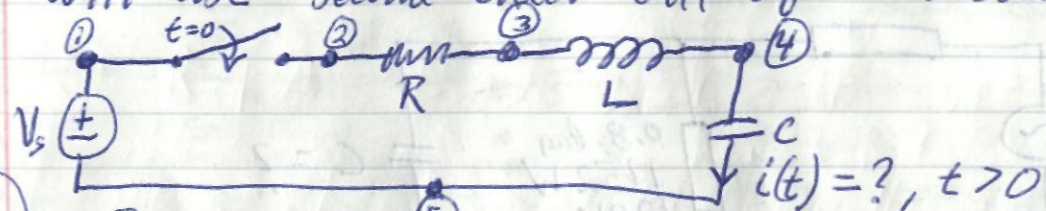
$$C = \frac{486}{120^2 \cdot 60 \cdot 2\pi} = 89.52 \mu\text{F}$$

$\omega = 2\pi f$
*Assume frequency is 60Hz unless otherwise noted.

$$a y'' + b y' + c y = 0$$

Second Order Circuits

These circuits have two energy storage elements
will use second order diff eq with constant coefficients



(in series)

$$V_R(t) + V_L(t) + V_C(t) = V_s$$

$$R i(t) + L \frac{di(t)}{dt} + \frac{1}{C} i(t) = 0$$

$$R \frac{di(t)}{dt} + L \frac{d^2 i(t)}{dt^2} + \frac{1}{C} i(t) = 0$$

$$\frac{d^2 i(t)}{dt^2} + \frac{R}{L} \frac{di(t)}{dt} + \frac{1}{LC} i(t) = 0$$

$$s^n \triangleq \frac{d^n}{dt^n}$$

$$s^2 + \frac{R}{L} s + \frac{1}{LC} = 0$$

$$\begin{aligned} V_R(t) &= R \cdot i(t) \\ V_L(t) &= L \cdot \frac{di(t)}{dt} \\ V_C(t) &= \frac{1}{C} \int i(t) dt \end{aligned}$$

- i. real, distinct Roots $\Rightarrow$ overdamped circuit
- ii. real, repeated Roots $\Rightarrow$ critically damped circuit
- iii. Complex conjugate pair of Roots $\Rightarrow$ under damped circuit

$$\omega_n = \sqrt{\frac{1}{LC}} \quad (\text{Zeta})_n = \frac{1}{2} R \sqrt{\frac{C}{L}}$$

$$x(t) = x_n(t) + x_f(t)$$

natural response forced response

$$V = IR$$

(Real distinct Roots) (overdamped)

$$s = s_1, s_2, s_1 \neq s_2$$

Know Memorize

$$* x(t) = \underbrace{\alpha_1 e^{s_1 t} + \alpha_2 e^{s_2 t}}_{x_n(t)} + \underbrace{x(\infty)}_{x_f(t)} \leftarrow \text{steady state response}$$

(Real Repeated Roots) (critically damped)

$$s = s_1 = s_2$$

$$* x(t) = \alpha_1 e^{s_1 t} + \alpha_2 t e^{s_1 t} + x(\infty)$$

(Complex Roots) (under damped)

$$s = -\zeta \pm j\omega_d$$

$$* x(t) = \alpha_1 e^{(-\zeta + j\omega_d)t} + \alpha_2 e^{(-\zeta - j\omega_d)t} + x(\infty)$$

(ex) $R = 5k\Omega, L = 1H, C = 1\mu F, V_s = 25V$

$$s^2 + \frac{5000}{1} s + \frac{1}{1} = 0$$

$$s^2 + 5000s + 1 = 0$$

$$s = -4791.23, -208.71 \rightarrow \text{overdamped}$$

$$i(t) = \alpha_1 e^{-208.71 t} + \alpha_2 e^{-4791.23 t} + i(\infty), i(\infty) = 0$$

$$0A = \alpha_1 e^{-0} + \alpha_2 e^{-0} \Rightarrow \alpha_1 = \alpha_2 \quad i(0) = 0$$

$$\frac{di(t)}{dt} = -208.71 \alpha_1 e^{-208.71 t} - 4791.23 \alpha_2 e^{-4791.23 t}, \frac{di(0)}{dt} = 7$$

$$Ri(0) + \frac{di(0)}{dt} + V_C(0) = V_s$$

$$5000 i(0) + \frac{di(0)}{dt} + V_C(0) = 25 \rightarrow \frac{di(0)}{dt} = 25 A/s$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$25 = -208.71 \alpha_1 e^{-208.71(0)} + 4791.23 \alpha_1 e^{-4791.23(0)}$$

$$\alpha_1 = 5.455 \Rightarrow \alpha_2 = -5.455$$

ex $R=1\Omega$ $L=0.5H$ $C=2F$ $V_s=25V$

$$S_1 = -1 = -1 = S_2 \Rightarrow \text{Real repeated roots, critically damped}$$

$$i(t) = \alpha_1 e^{-t} + \alpha_2 t e^{-t} + i(\infty), i(\infty) = 0$$

$$i(t) = \alpha_1 e^{-t} + \alpha_2 t e^{-t} \quad i(0) = 0$$

$$0 = \alpha_1 e^{-0} + \alpha_2 0 e^{-0} \quad \alpha_1 = 0$$

$$i(t) = \alpha_2 t e^{-t}$$

$$\frac{di(t)}{dt} = \alpha_2 e^{-t} - \alpha_2 t e^{-t}, \quad \frac{di(0)}{dt} = V_c(0) = 0V$$

$$Ri(0) + \frac{di(0)}{dt} + V_c(0) = V_s$$

$$i(0) + 0.5 \frac{di(0)}{dt} + 0 = 25 \Rightarrow \frac{di(0)}{dt} = 50 A/s$$

$$50 = \alpha_2 e^{-0} - \alpha_2 0 e^{-0} \Rightarrow \alpha_2 = 50$$

$$i(t) = 50 t e^{-t} A, t > 0$$

ex $R=200\Omega$ $L=0.5H$ $C=10\mu F$ $V_s=25V$

$$S_1 = -200 - j400 \quad \text{complex roots underdamped}$$

$$S_2 = -200 + j400$$

$$i(t) = \alpha_1 e^{(-200+j400)t} + \alpha_2 e^{(-200-j400)t} + i(\infty) \quad i(\infty) = 0$$

$$0 = \alpha_1 e^{(-200+j400)0} + \alpha_2 e^{(-200-j400)0} \quad i(0) = 0$$

$$\Rightarrow \alpha_1 = -\alpha_2$$

$$\frac{di(t)}{dt} = (-200+j400)\alpha_1 e^{(-200+j400)t} + (200+j400)\alpha_1 e^{(-200-j400)t}$$

$$Ri(0) + \frac{di(0)}{dt} + V_c(0) = V_s$$

$$200 i(0) + 0.5 \frac{di(0)}{dt} + 0V = 25 \Rightarrow \frac{di(0)}{dt} = 50 A/s$$

complex roots $\rightarrow$ underdamped must be simplified in this way

$$10 \quad 50 = (-200+j400)\alpha_1 e^{(-200+j400)0} + (200+j400)\alpha_1 e^{(-200-j400)0}$$

$$11 \quad \alpha_1 = -j0.0625 \Rightarrow \alpha_2 = j0.0625$$

$$i(t) = -j0.0625 e^{(-200+j400)t} + j0.0625 e^{(-200-j400)t} A, t > 0$$

$$i(t) = 0.0625 (-j e^{(-200+j400)t} + j e^{(-200-j400)t})$$

$$i(t) = 0.0625 (-j e^{-200t} e^{j400t} + j e^{-200t} e^{-j400t})$$

$$i(t) = 0.0625 e^{-200t} (-j e^{j400t} + j e^{-j400t})$$

$$e^{\pm jx} = \cos(x) \pm j \sin(x)$$

$$\cos(x) = \frac{e^{jx} + e^{-jx}}{2}$$

$$\sin(x) = \frac{e^{jx} - e^{-jx}}{j2}$$

$$i(t) = 0.0625 e^{-200t} [-j(\cos(400t) + j \sin(400t)) + j(\cos(400t) - j \sin(400t))]$$

$$i(t) = 0.0625 e^{-200t} [-j^2 2 \sin(400t)]$$

$$i(t) = 0.125 e^{-200t} \sin(400t) A, t > 0$$

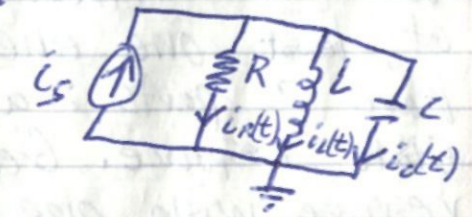
Everything in Parallel Topology

$$x(t) = \alpha_1 e^{s_1 t} + \alpha_2 t e^{s_1 t} + x(\infty)$$

same equations for each root combination



after switch action $\rightarrow$



$$i_R(t) + i_L(t) + i_C(t) = i_s \quad \leftarrow KCL$$

$$\frac{v(t)}{R} + \frac{1}{L} \int_{-\infty}^t v(\tau) d\tau + C \frac{dv(t)}{dt} = i_s$$

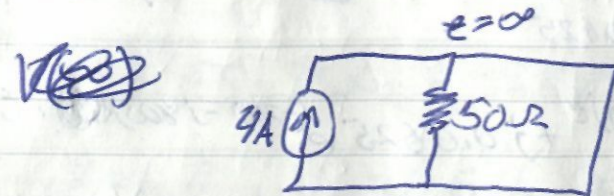
$$\frac{1}{R} \frac{dv(t)}{dt} + \frac{1}{L} v(t) + C \frac{d^2 v(t)}{dt^2} = 0$$

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

$\leftarrow$ characteristic equation for everything in parallel topology

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

ex) $R = 50\Omega$ $L = 100H$ $C = 10mF$ $i_s = 4A$



$$V(\infty) = 0V$$

$$V(0) = 0V$$

$$s^2 + 2s + 1 = 0 \rightarrow s = -1, -1 \Rightarrow \text{critically damped}$$

$$V(t) = \alpha_1 e^t + \alpha_2 t e^t + V(\infty)$$

$$V(\infty) = 0V$$

$$V(t) = \alpha_1 e^{-t} + \alpha_2 t e^{-t} \text{ and } V(0) = 0V$$

$$0V = \alpha_1 e^{-0} + \alpha_2(0)e^{-0} \Rightarrow \alpha_1 = 0$$

$$V(t) = \alpha_2 t e^{-t}$$

$$\frac{dV(t)}{dt} = \alpha_2 e^{-t} - \alpha_2 t e^{-t}$$

$$\frac{V(0)}{50} + i_L(0) + 0.01 \frac{dV(0)}{dt} = 4A \Rightarrow \frac{dV(0)}{dt} = 400 V/s$$

$$400 V/s = \alpha_2 e^{-0} - \alpha_2(0)e^{-0} \Rightarrow \alpha_2 = 400$$

$$V(t) = 400 t e^{-t} V, t > 0$$

AC Analysis

We know the response of a circuit which contains at least one energy storage element (L or C) will have 2 parts: a transient response and a steady state response. Generally speaking, finding the transient response with one L or C and a sinusoidal source is very difficult and of limited use. Thus, we will concentrate upon finding only the steady state response aka the forced response. Finding the steady state response with sinusoidal source is called AC steady-state analysis

2 Observations

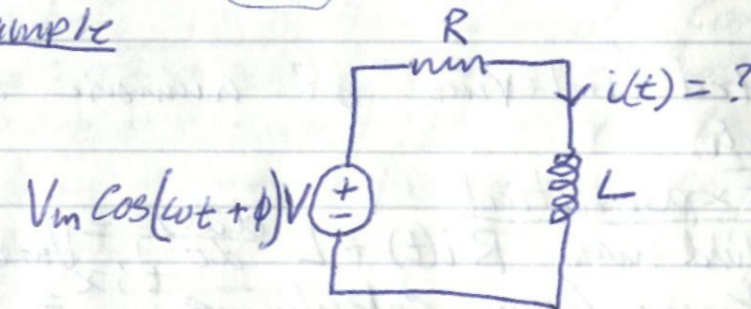
1. We know KVL and KCL must be satisfied in a circuit regardless of the type of source

2. If we apply a sinusoidal source to a linear circuit, then KVL and KCL require all voltages and currents within the circuit to be sinusoids of the same frequency. We will see momentarily, there are 2 parts to the AC steady state analysis

Part I: examine the circuit & setup the D.E. which describes the desired voltage or current.

Part II: Solve the D.E. (Observation 1 helps with part I; observation 2 helps with part II.)

Example



Part I: Setup the D.E.

By KVL: $V_R(t) + V_L(t) = V_s(t)$

$$Ri(t) + L \frac{di(t)}{dt} = V_m \cos(\omega t + \phi), \text{ let } \phi = 0 \text{ (for ease)}$$

Part II: Solve the D.E. We have 3 options for solving the D.E.: brute force, complex exponential, and the geometric approach.

The Brute Force Method (BFM)

as mentioned earlier, $i(t)$ must be a sinusoid of the same frequency as the voltage source; however, $i(t)$ may have a different amplitude & phase angle.

Thus, $i(t) = A \cos(\omega t + \theta)$, where A & θ are unknown.

$$R(A \cos(\omega t + \theta)) + L \frac{d}{dt}(A \cos(\omega t + \theta)) = V_m \cos(\omega t)$$

$$RA \cos(\omega t + \theta) - LA \omega \sin(\omega t + \theta) = V_m \cos(\omega t)$$

$$RA \{ \cos(\theta) \cos(\omega t) - \sin(\theta) \sin(\omega t) \} - LA \omega \{ \sin(\theta) \cos(\omega t) + \cos(\theta) \sin(\omega t) \} = V_m \cos(\omega t)$$

$$[RA \cos(\theta) - LA \omega \sin(\theta)] \cos(\omega t) - [RA \sin(\theta) + LA \omega \cos(\theta)] \sin(\omega t) = V_m \cos(\omega t)$$

missing
40

$$RA \cos(\theta) - LA\omega \sin(\theta) = V_m$$

$$- [RA \sin(\theta) + LA\omega \cos(\theta)] = 0$$

We have a system of equations, solving simultaneously is lots of hassle & need to use lots of trig ids

$$A = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \text{ and } \theta = \arctan\left(\frac{-\omega L}{R}\right), \text{ which implies...}$$

$$i(t) = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \cos(\omega t + \arctan\left(\frac{-\omega L}{R}\right)) A.$$

This is complicated (in terms of numeric manipulation) go to next approach

The Complex Exponential

The D.E. we solved was $Ri(t) + L \frac{di(t)}{dt} = V_m \cos(\omega t)$. Consider this: we know from calculus $e^{\pm jx} = \cos(x) \pm j \sin(x)$ while this is not intuitive one

proves it easily. What good is this? Suppose the source was $V_m e^{j\omega t}$ rather than $V_m \cos(\omega t)$. Would the D.E. be more easily solved? The D.E. has become $Ri(t) + L \frac{di(t)}{dt} = V_m e^{j\omega t}$, and it is more easily solved than the original D.E. because the answer will be in exponential form

rather than in trig form. The exponentials are easier to manipulate. Let's solve the D.E.

We also know the solution is of the form $i(t) = A e^{j(\omega t + \theta)}$, where A & θ are unknowns.

$$RA e^{j(\omega t + \theta)} + L \frac{d}{dt}(A e^{j(\omega t + \theta)}) = V_m e^{j\omega t}$$

$$RA e^{j(\omega t + \theta)} + j\omega L A e^{j(\omega t + \theta)} = V_m e^{j\omega t}$$

$$A(R + j\omega L) e^{j(\omega t + \theta)} = V_m e^{j\omega t}$$

$$A(R + j\omega L) e^{j\omega t} e^{j\theta} = V_m e^{j\omega t}$$

$$A(R + j\omega L) e^{j\theta} = V_m$$

Now we have a complex equation w/ 2 unknowns we can solve it by setting the real parts of the

LHS & RHS equal to each other & the imaginary parts of the LHS & RHS to each other. First, do some manipulation

$$A e^{j\theta} = \frac{V_m}{R + j\omega L}$$

We may solve this in two ways: use Euler's identity to express the complex exponentials as sines and cosine

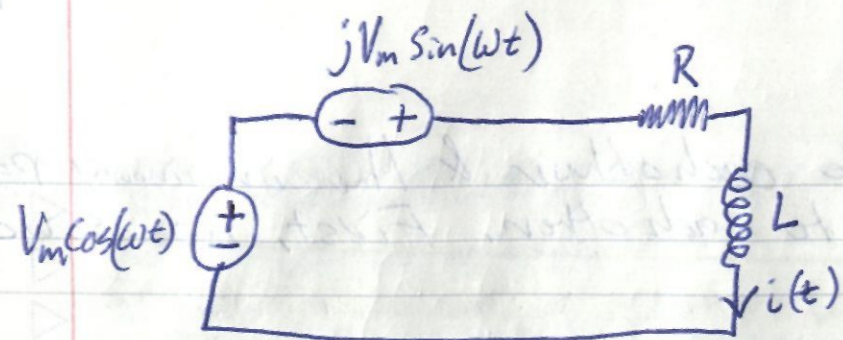
or we can express the LHS & RHS as magnitudes and phases. The second method will be easier. A is the magnitude, θ is the phase. Convert $R + j\omega L$ to polar form: $\sqrt{R^2 + (\omega L)^2} \angle \arctan \frac{\omega L}{R}$. $A e^{j\theta}$ becomes $A \angle \theta$ because $e^{j\theta} = \cos \theta + j \sin \theta = 1 \angle \theta \Rightarrow (A)(1 \angle \theta) = A \angle \theta$. V_m becomes $V_m \angle 0^\circ$ since it is purely real. We now have...

$$A \angle \theta = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \angle -\arctan\left(\frac{\omega L}{R}\right) \text{ we can claim...}$$

$$A = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \text{ and } \theta = -\arctan\left(\frac{\omega L}{R}\right).$$

$$i(t) = \left(\frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \right) e^{j(\omega t + \arctan(-\frac{\omega L}{R}))} A.$$

We now see why the complex exponential is easier. We can multiply and divide complex numbers more easily: the unknowns are isolated. This is nice, but we do not have sources like $V_m e^{j\omega t}$ in the real world, instead we have sinusoidal sources. So what good are complex exponentials in the real world? We know $e^{jx} = \cos(x) + j \sin(x)$, so $V_m e^{j\omega t} = V_m \cos(\omega t) + j V_m \sin(\omega t)$. The circuit diagram becomes... (next page)

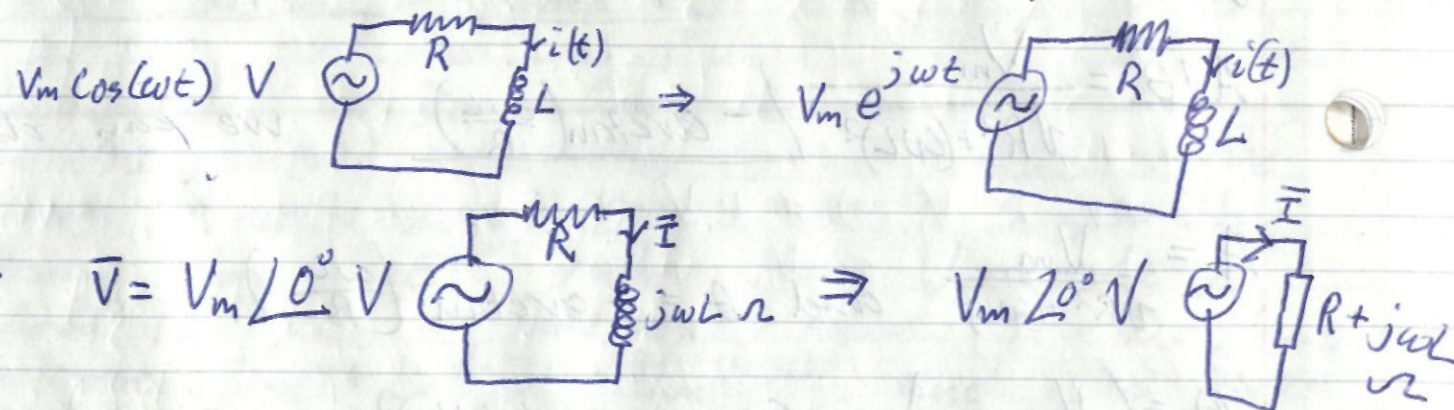


$i(t)$ is found by
Superposition. $i(t)$
will have a real
part & an imaginary part. We can solve the problem
by: replacing the sources with a complex exponential; Solving
the new D.E. and disregarding the imaginary part.

$$i(t) = \left(\frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \right) \cos(\omega t + \arctan(-\frac{\omega L}{R})) A$$

The Geometric Approach

Consider the original circuit using Phasors and impedances.



Recall, $\bar{V} = \bar{I} \bar{Z} \Rightarrow \bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{V_m / 0^\circ}{R + j\omega L} \Rightarrow$
 $\bar{I} = \frac{V_m / 0^\circ}{\sqrt{R^2 + (\omega L)^2} \angle \arctan(\frac{\omega L}{R})} \Rightarrow \bar{I} = \frac{V_m}{\sqrt{R^2 + \omega L^2}} \angle \arctan(-\frac{\omega L}{R}) A$

disregarding the imaginary part...

$$i(t) = \left(\frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \right) \cos(\omega t + \arctan(-\frac{\omega L}{R})) A$$

N.b. we no longer require a D.E.

The Laplace Transform

The Laplace transform converts a signal from the time domain to the frequency domain.

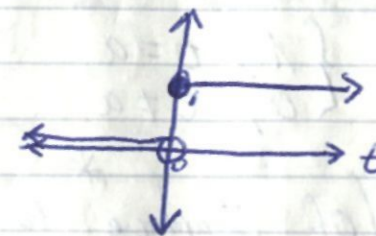
$$F(s) = \mathcal{L}\{f(t)\} = \int_{-\infty}^{\infty} f(t) e^{-st} dt \quad (\text{Bilateral})$$

$$F(s) = \int_0^{\infty} f(t) e^{-st} dt \quad (\text{unilateral})$$

The Step Function

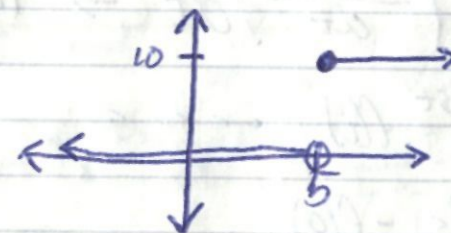
$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

" $u(t)$ turns on @ $t=0$ "



$u(t)$ may be shifted to turn on @ $t=a \Rightarrow u(t-a)$ (time delay)
 $t=-a \Rightarrow u(t+a)$ (time advance)

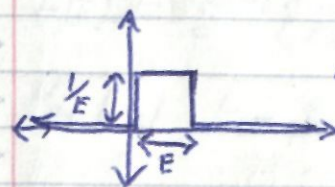
(ex) $10u(t-5)$



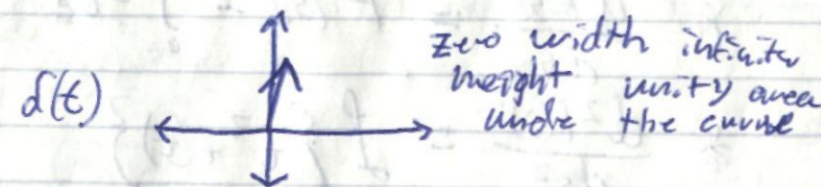
The Impulse Function

$\delta(t)$ aka the Dirac delta function $\delta(t)$ defines a derivative @ a discontinuity - i.e. the stepfunction

$$\delta(t) = \frac{du(t)}{dt}$$



$A=1$



$$\delta(t) = \begin{cases} 1, & t=0 \\ 0, & t \neq 0 \end{cases}$$

An area under the curve other than unity, say K , means the strength is K and one has $K\delta(t)$

$$\Rightarrow f(t) = \begin{cases} K, & t=0 \\ 0, & t \neq 0 \end{cases}$$

$$f(t) = \int_{-\infty}^{\infty} K \delta(t) dt = K$$

$$f(t-a) = \begin{cases} 1, & t=a \\ 0, & t \neq a \end{cases}$$

Properties* (of Laplace to memorize)*

Linearity: $\mathcal{L}\{K f(t)\} = K F(s)$

$$\mathcal{L}\{f_1(t) + f_2(t) + \dots + f_n(t)\} = \mathcal{L}\{f_1(t)\} + \mathcal{L}\{f_2(t)\} + \dots + \mathcal{L}\{f_n(t)\}$$

$$\mathcal{L}\left[\frac{df(t)}{dt}\right] = \int_0^{\infty} e^{-st} \left(\frac{df(t)}{dt}\right) dt \rightarrow \text{integrate by parts}$$

$$dv = \left(\frac{df(t)}{dt}\right) dt \rightarrow e^{-st} f(t) \dots$$

$$* \mathcal{L}\left\{\frac{df(t)}{dt}\right\} = sF(s) - f(0)$$

$$* \mathcal{L}\left\{\frac{d^2 f(t)}{dt^2}\right\} = s^2 F(s) - sf(0) - f'(0)$$

$$* \frac{dF(s)}{ds} = F'(s) = \int_0^{\infty} -te^{-st} f(t) dt = \mathcal{L}\{-tf(t)\}$$

$$* \frac{d^2 F(s)}{ds^2} = \mathcal{L}\{t^2 f(t)\}$$

$$* \frac{d^{(n)} F(s)}{ds^n} = \mathcal{L}\{(-t)^n f(t)\}$$

$$* \mathcal{L}\left\{\int_0^t f(x) dx\right\} = \frac{F(s)}{s}$$

Time Domain Translation: $\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as} F(s)$

Frequency Domain Translation: $F(s+a) = \mathcal{L}\{e^{-at} f(t)\}$

Scaling: $\mathcal{L}\{f(ct)\} = \frac{1}{c} F\left(\frac{s}{c}\right), c > 0$

$$\text{Convolution: } y(t) = x(t) \otimes h(t) = \int_0^{\infty} x(\tau) h(t-\tau) d\tau = \int_0^{\infty} x(t-\tau) h(\tau) d\tau$$

$$\text{Transforms } \mathcal{L}\{u(t)\} = \int_0^{\infty} e^{-st} u(t) dt = \int_0^{\infty} e^{-st} dt = \frac{1}{s}, s > 0$$

$$\left[-se^{-st}\right]_0^{\infty} = -se^{-s\infty} - se^{-0}$$

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} f(t) e^{-st} dt = 1$$

$$\mathcal{L}\{f'(t)\} = s \text{ and } \mathcal{L}\{f^{(n)}(t)\} = s^n$$

$$\mathcal{L}\{e^{-at}\} = \int_0^{\infty} e^{-at} e^{-st} dt = \frac{1}{s+a}$$

$$\mathcal{L}\{\sin(\omega t)\} = \int_0^{\infty} e^{-st} \sin(\omega t) dt = \frac{\omega}{s^2 + \omega^2}$$

$$\mathcal{L}\{\cos(\omega t)\} = \int_0^{\infty} e^{-st} \cos(\omega t) dt = \frac{s}{s^2 + \omega^2}$$

$$\text{to solve use } \cos(x) = \frac{e^{jx} + e^{-jx}}{2} \quad \sin(x) = \frac{e^{jx} - e^{-jx}}{j2}$$

$$\text{Inverse Laplace Transform } f(t) = \mathcal{L}^{-1}\{F(s)\} = \frac{1}{j2\pi} \int_{-\infty}^{\infty} F(s) e^{st} ds$$

$$\int_0^{\infty} e^{-st} \cdot \frac{e^{j\omega t} + e^{-j\omega t}}{2} dt \quad \int_0^{\infty} \frac{1}{2} e^{\omega t} \cdot e^{-st} dt$$

Partial Fraction Expansion (PFE)

$$F(s) = \frac{N(s)}{D(s)} = \frac{a_n s^n + a_{n-1} s^{n-1} + \dots + a_0 s^0}{b_m s^m + b_{m-1} s^{m-1} + \dots + b_0 s^0}$$

$m > n \rightarrow$ proper rational function
 $m \leq n \rightarrow$ improper rational function

Proper Rational Functions ($m < n$)

→ ~~Be~~ Be sure $k_m = 1$

① Factor $D(s)$.

② → real, distinct roots

③ → real, repeated roots

④ → complex conjugate pair of roots

a. Real Distinct Roots

ex

$$F(s) = \frac{s+10}{s^3+7s^2+6s} = \frac{s+10}{s(s+1)(s+6)}$$

$$= \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+6} = \frac{s+10}{s(s+1)(s+6)}$$

$$\cancel{A} + \cancel{\frac{B}{s+1}} + \frac{C}{s+6} = \cancel{\frac{s+10}{s(s+1)(s+6)}}$$

$$A = 1.6667 \quad B = -1.8 \quad C = 0.1333$$

Figure this out!!

$$F(s) = \frac{1.6667}{s} + \frac{-1.8}{s+1} + \frac{0.1333}{s+6}$$

inverse transforming

$$f(t) = 1.6667 u(t) - 1.8e^{-t} + 0.1333e^{-6t}$$

Real Repeated Roots

$$F(s) = \frac{s+10}{(s)(s+6)(s+2)^2} = \frac{A}{s} + \frac{B}{s+6} + \frac{C}{s+2} + \frac{D}{(s+2)^2}$$

$$s+10 = \cancel{(s+6)(s+2)^2} A + (s)(s+2)^2 B + (s)(s+6)(s+2) C + (s)(s+6) D$$

$$(s+2)^2((s+6)A + (s)B)$$

$$A = 0.4167$$

$$B = -0.04167$$

$$D = -1$$

to find C multiply both sides by that root raised to the higher power. then take the ~~derivative~~ of both sides respect to s . then evaluate both sides ~~at~~ at $s = \text{the root (this case } s = -2)$

$$\frac{s+10}{s(s+6)} = \frac{(s+2)^2 \cdot 0.4167}{s} + \frac{(s+2)^2 \cdot -0.04167}{s+6} + (s+2)C - 1$$

$$\frac{s+10}{s^2+6s} = \frac{(s^2+4s+4) \cdot 0.4167}{s} + \frac{(s^2+4s+4) \cdot -0.04167}{s+6} + (s+2)C - 1$$

$$\frac{1}{s+6} + \frac{10}{s^2+6s} = s \cdot 0.4167 + 4 \cdot 0.4167$$

$$F(s) = \frac{s+10}{(s)(s+6)(s+2)^2} = \frac{0.4167}{s} + \frac{-0.04167}{s+6} + \frac{-0.375}{s+2} - \frac{1}{(s+2)^2}$$

$$f(t) = 0.4167 u(t) - 0.04167 e^{-6t} - 0.375 e^{-2t} - t e^{-2t}$$

Complex Conjugate Pair of Roots

$$F(s) = \frac{s+10}{s^3+6s^2+25s} = \frac{s+10}{(s)(s+3+j4)(s+3-j4)}$$

$$\frac{s+10}{(s)(s+3+j4)(s+3-j4)} = \frac{A}{s} + \frac{B}{s+3+j4} + \frac{C}{s+3-j4}$$

$$s+10 = A(s+3+j4)(s+3-j4) + B(s)(s+3-j4) + C(s)(s+3+j4)$$

Solve for A, B, C

$$A = 0.4$$

$$B = 0.2016 \angle -172.88^\circ$$

$$C = 0.2016 \angle 17.12^\circ$$

$$f(t) = 0.4 u(t) + 0.2016 \angle -172.88^\circ e^{-(3+j4)t} + 0.2016 \angle 172.88^\circ e^{-(3-j4)t}$$

$$f(t) = 0.4 u(t) + 0.2016 e^{-j172.88^\circ} e^{-(3+j4)t} + 0.2016 e^{j172.88^\circ} e^{-(3-j4)t}$$

$$f(t) = 0.4 u(t) + 0.2016 (e^{-j172.88^\circ} e^{-3t-j4t} + e^{j172.88^\circ} e^{-3t+j4t})$$

$$f(t) = 0.4 u(t) + 0.2016 e^{-3t} (e^{-j172.88^\circ} e^{-j4t} + e^{j172.88^\circ} e^{j4t})$$

$$f(t) = 0.4 u(t) + 0.2016 e^{-3t} (e^{-j(4t+172.88^\circ)} + e^{j(4t+172.88^\circ)})$$

$$f(t) = 0.4 u(t) + 0.2016 e^{-3t} (2 \cos(4t + 172.88^\circ))$$

$$f(t) = 0.4 u(t) + 0.2016 e^{-3t} (2 \cos(4t + 172.88^\circ))$$

$$f(t) = 0.4 u(t) + 0.2016$$

$$f(t) = 0.4 u(t) + 0.4032 e^{-3t} \cos(4t + 172.88^\circ)$$

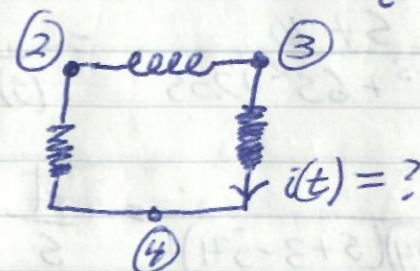
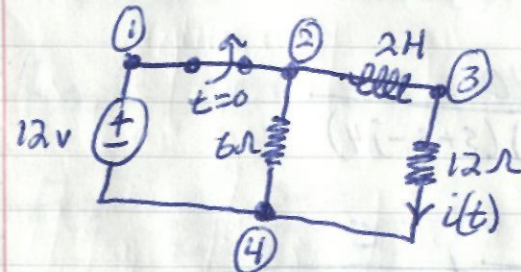
Improper Rational Function ($m \leq n$)

divide the numerator by the denominator. The remainder will be a proper rational function. Then use PFE on the remainder

Initial and Final Value Theorems

initial value Theorem (IVT)

Final value Theorem (FVT)



$$6 i(t) + 2 \frac{di(t)}{dt} + 12 i(t) = 0$$

$$18 i(t) + 2 \frac{di(t)}{dt} = 0 \rightarrow 18 I(s) + 2(s I(s) - i(0)) = 0$$

$$18 I(s) + 2s I(s) - 2i(0) = 0$$

$$I(s)(18 + 2s) = 2i(0)$$

$$i(0) = 1A \text{ by inspection}$$

$$I(s) = \frac{2 i(0)}{18 + 2s}$$

$$I(s) = \frac{2}{2(9+s)} = \frac{1}{9+s}$$

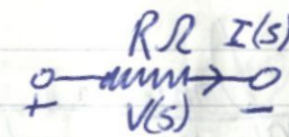
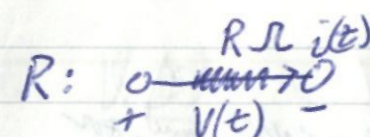
$$i(t) = e^{-9t} A, t > 0$$

IVT

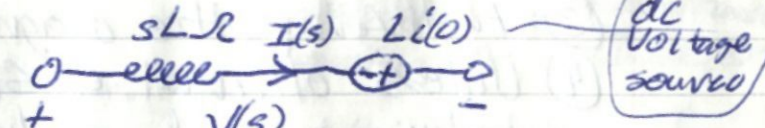
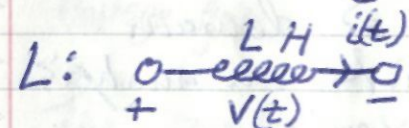
FVT

$$\frac{1}{L} \int_{-\infty}^t v(\tau) d\tau \rightarrow \frac{1}{L} \frac{V(s)}{s} \text{ or } \frac{V(s)}{sL}$$

* Transforming circuit elements * memorize



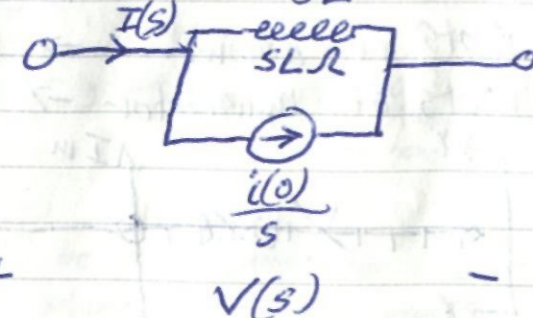
$$\mathcal{L}\{V(t)\} = \mathcal{L}\{R \cdot i(t)\} \leftrightarrow V(s) = R \cdot I(s)$$



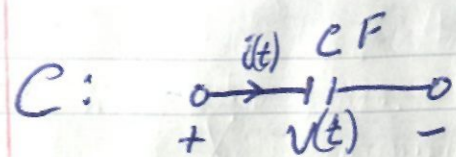
$$\mathcal{L}\{v(t)\} = \mathcal{L}\{L \cdot \frac{di(t)}{dt}\} \leftrightarrow V(s) = sL I(s) - L i(0)$$

$$\frac{V(s) + L i(0)}{sL} = I(s)$$

$$I(s) = \frac{V(s)}{sL} + \frac{i(0)}{s}$$

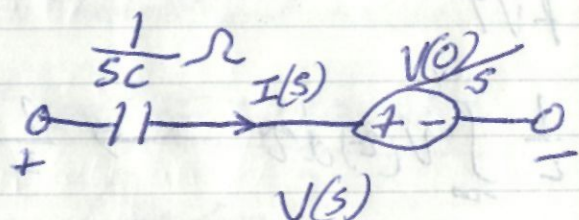


Formula $\rightarrow \frac{\square}{\square}$



$$\mathcal{L}\{i(t)\} = \mathcal{L}\left\{C \frac{dv(t)}{dt}\right\} \leftrightarrow I(s) = sC V(s) - C v(0)$$

$$V(s) = \frac{I(s)}{sC} + \frac{v(0)}{s}$$



$s = \sigma + j\omega$, let $\sigma = 0$
 $\rightarrow s = j\omega$

S Domain Process

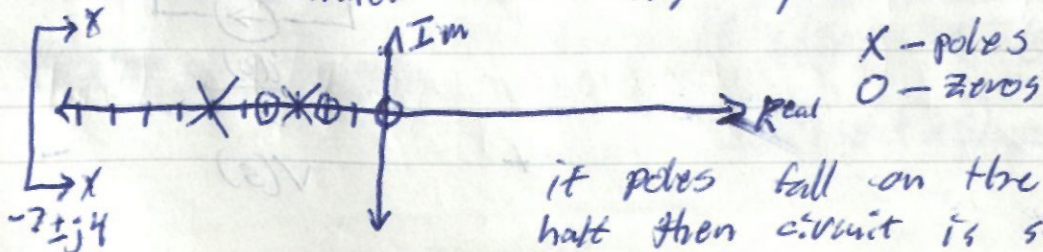
- 1) Label your nodes
- 2) note the initial conditions - for each inductor and capacitor
- 3) transform the diagram to the S domain
- 4) Unless otherwise specified pick an analysis technique and analyze the circuit.

then we do blank examples

A Word About Poles and Zeros

Consider $F(s) = \frac{(s+2)(s+4)(s)}{(s+3)(s+6)(s+7+j4)(s+7-j4)}$

- (Poles) roots of denom. $\rightarrow s = -3, -6, (-7 \pm j4)$
 (Zeros) roots of numerator $\rightarrow s = -2, -4, 0$



if poles fall on the left half then circuit is stable (not blow up)

if just one pole is on the right side then circuit is unstable $\rightarrow$ (will blow up)

if poles are on left half and even just one is on the axis then it is critically stable (don't know if it will blow up or not)

The Transfer Function

$y(t) = x(t) \otimes h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$
 Convolution

go to S domain

$Y(s) = X(s) H(s)$
 $H(s) = \frac{Y(s)}{X(s)}$

all i.c.s must be equal to zero in order to determine $H(s)$.

Four Kinds of Transfer Functions

i. $\frac{V_o(s)}{V_i(s)}$

Voltage gain

ii. $\frac{I_o(s)}{I_i(s)}$

Current gain

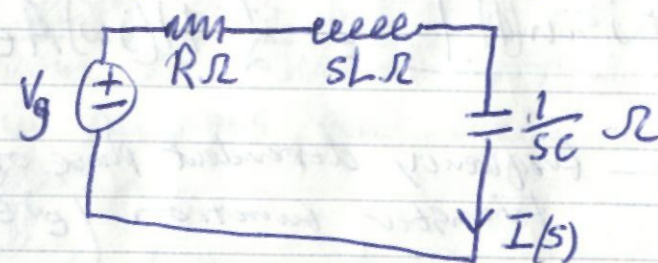
iii. $\frac{V_o(s)}{I_i(s)}$

Impedance transfer

iv. $\frac{I_o(s)}{V_i(s)}$

admittance transfer

The input and output signals may be arbitrarily defined.



$H(s) = \frac{I(s)}{V_g}$

$I(s) = \frac{V_g}{R + sL + \frac{1}{sC}}$

$H(s) = \frac{V_g}{R + sL + \frac{1}{sC}} \Rightarrow H(s) = \frac{1}{R + sL + \frac{1}{sC}}$

in this course don't simplify from here

$\mathcal{L}^{-1}\{H\} = h(t)$ (note) the poles must be restricted to the left half of complex plane; the zeros may occur anywhere on the complex plane.

When you take the $\mathcal{L}^{-1}$ of $Y(s)$, $H(s)$ generates the transient response, $X(s)$ generates the steady-state response. $H(s)$ & $X(s)$ accomplish this by their poles

$$x(t) = \delta(t) \leftarrow \text{impulse function}$$

$$X(s) = 1$$

$$Y(s) = H(s) X(s)$$

$$Y(s) = H(s)$$

$$y(t) = h(t) \leftarrow \text{impulse response}$$

Sinusoidal Steady-State Response

$$x(t) = A \cos(\omega t + \phi)$$

$$x(t) = A \cos(\omega t) \cos(\phi) - A \sin(\omega t) \sin(\phi)$$

$$X(s) = \frac{(A \cos(\phi))s}{s^2 + \omega^2} - \frac{(A \sin(\phi))\omega}{s^2 + \omega^2}$$

$$Y(s) = \frac{K}{s - j\omega} + \frac{K^*}{s + j\omega} + \sum (\text{poles of } H(s))$$

$$K = \frac{[H(s)](A)[s \cos(\phi) - \omega \sin(\phi)]}{s + j\omega} \Big|_{s = j\omega} \quad (\text{evaluate})$$

$$K = \frac{[H(j\omega)](A)[\cos(\phi) + j \sin(\phi)]}{2} = \frac{1}{2} H(j\omega) A e^{j\phi}$$

$$H(j\omega) = \underbrace{|H(j\omega)|}_{\text{frequency dependant magnitude of transfer function}} e^{j\theta(\omega)} \leftarrow \text{frequency dependant phase of transfer function } (e^{j\theta(\omega)})$$

$$K = \frac{A}{2} |H(j\omega)| e^{j\theta(\omega)} e^{j\phi} \Rightarrow K = \frac{A}{2} |H(j\omega)| e^{j(\theta(\omega) + \phi)}$$

$$y_{ss}(t) = A |H(j\omega)| \cos(\omega t + \theta(\omega) + \phi)$$

Vocab

- a pass band - the range of frequencies where the ratio of output power to input power is at its maximum.
- a stop band is the range of frequencies where the ratio of output power to input power is not at its maximum.

Frequency Response Plot - aka Bode plots

↳ magnitude plot

↳ phase plot

↳ Cutoff frequency aka half-power point aka 3dB powerpoint

- a cutoff frequency is the frequency where the ratio of output power to input power is 0.5.

↑ (thus half power point)

- a low pass filter is when you have a pass band from zero radians per second to the cutoff frequency and a stop band from the cutoff frequency to infinite radians per second.

- a high pass filter is when you have a stop band from zero radians per second to the cutoff frequency and a pass band from the cutoff frequency to infinite rad/s.

- a band pass filter is when you have a stop band from zero radians per sec, to the first cutoff frequency, a pass band from the first cutoff frequency to second cutoff frequency, and a stop band from the second cutoff frequency to ∞ rad/s.

aka band reject aka notch filter

- a band stop filter is when you have a pass band from zero rad/s to the first cutoff frequency, a stop band from the first cutoff frequency to the second cutoff frequency, and a pass band from the second cutoff frequency to ∞ rad/s.

Doing Bode Plots by Hand

- get the transfer function in standard form
- write the magnitude and phase expressions (polar form)
- Draw the plots

getting to Standard Form: $H(s) = \frac{K(s+z_1)}{(s+p_1)(s+p_2)}$ fully factored

$= \frac{Kz_1(\frac{s}{z_1} + 1)}{(s)(p_1)(\frac{s}{p_1} + 1)}$ let $K_0 = \frac{Kz_1}{p_1} \Rightarrow H(s) = \frac{K_0(\frac{s}{z_1} + 1)}{(s)(\frac{s}{p_1} + 1)}$ let $s = j\omega$

$\Rightarrow H(j\omega) = \frac{K_0(\frac{j\omega}{z_1} + 1)}{(j\omega)(\frac{j\omega}{p_1} + 1)}$ ← now in standard form

now put in polar form

$|H(j\omega)| = \frac{|K_0| |\frac{j\omega}{z_1} + 1|}{|\omega| |\frac{j\omega}{p_1} + 1|}$

$H(s) = \frac{V(s)}{V_i(s)}$ $p = \frac{V^2}{R}$

$\theta(\omega) = 0^\circ + \arctan\left(\frac{\omega}{z_1}\right) - 90^\circ$

$\theta(\omega) = 0^\circ + \arctan\left(\frac{\omega}{z_1}\right) - 90^\circ$
 ← arctan $\left(\frac{\omega}{p_1}\right)$ ← curve from denominator

$H(s) = \frac{(V_o(s)/R)}{(V_i(s)/R)} = H(s) = \frac{V_o(s)}{V_i(s)}$

$20 \log_{10}(|H(s)|) = 20 \log_{10}\left(\frac{V_o(s)}{V_i(s)}\right)$

$20 \log_{10}(|H(j\omega)|) = 20 \log_{10}(|K_0|) + 20 \log_{10}\left|\frac{j\omega}{z_1} + 1\right| - 20 \log_{10}|\omega| - 20 \log_{10}\left|\frac{j\omega}{p_1} + 1\right|$

First-order poles: -ive slope @ 20 dB/decade
 Nth-order poles: - slope @ 20N dB/decade

First-order zeros: + slope @ 20 dB/decade
 Nth-order zeros: + slope @ 20N dB/decade

+ terms in $\theta(\omega)$: + slope @ 45°/dec
 - terms in $\theta(\omega)$: - slope @ 45°/dec

The Band-Pass Filter
 5 characteristic values

- Center/resonant frequency, ω_0
 - ω_{c1} or ω_1
 - ω_{c2} or ω_2
 - bandwidth, β
 - Q, Quality Factor
- $\omega_0 = \sqrt{\omega_1 \omega_2}$ (don't use in this course)

$j\omega_0 L + \frac{1}{j\omega_0 C} = 0$

series RLC topology → $\omega_0 = \frac{1}{\sqrt{LC}}$, an exact value in (Rad/s) (topology specific)

$\omega_0 = 2\pi f_0 \Rightarrow f_0 = \frac{\omega_0}{2\pi}$ Hz

Memorize

$$\omega_1 = \frac{-\frac{R}{L} + \sqrt{\left(\frac{R}{L}\right)^2 + 4\omega_0^2}}{2} \text{ rad/s}$$

$$\omega_2 = \frac{\frac{R}{L} + \sqrt{\left(\frac{R}{L}\right)^2 + 4\omega_0^2}}{2} \text{ rad/s}$$

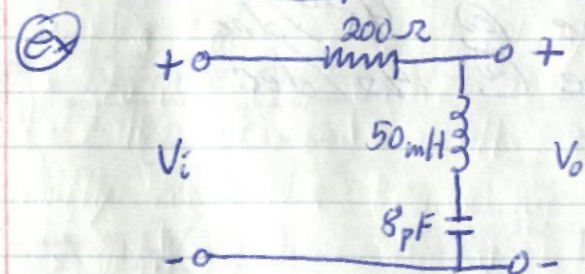
band width $\beta = \omega_2 - \omega_1$ not topology specific ✓

$$\frac{\frac{R}{L} + \sqrt{\left(\frac{R}{L}\right)^2 + 4\omega_0^2}}{2} - \frac{-\frac{R}{L} + \sqrt{\left(\frac{R}{L}\right)^2 + 4\omega_0^2}}{2}$$

Memorize $\beta = \frac{R}{L} \text{ Rad/s}$ topology specific

Memorize $Q = \frac{\omega_0}{\beta}$ Not Topology Specific (unitless) ✓

Band-Stop Filter



ω_0
 ω_1
 ω_2
 β
 Q

Same as before because we did not change component values or topology

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{(sL + \frac{1}{sC})}{(sL + R + \frac{1}{sC})} V_i(s)$$

$$H(s) = \frac{(sL + \frac{1}{sC})}{(sL + R + \frac{1}{sC})} \cdot \frac{s}{s} = \frac{(sL + \frac{1}{sC})}{(sL + R + \frac{1}{sC})} \cdot \frac{s}{s}$$

$$H(s) = \frac{(s^2 L + \frac{1}{C})}{(s^2 L + Rs + \frac{1}{C})} \xrightarrow{\text{sub values}} H(s) = \frac{s^2 + 2.5E12}{(s^2 + 0.1s + 2.5E12)}$$

$$H(s) = \frac{4E-13s^2 + 1}{(\frac{s}{\omega_1} + 1)(\frac{s}{\omega_2} + 1)} \rightarrow |H(j\omega)| = \frac{|1 - 4E-13\omega^2|}{\left|\frac{j\omega}{1.579E6} + 1\right| \left|\frac{j\omega}{1.583E6} + 1\right|}$$

also topology specific to Series RLC

$$|H(j\omega)| = \frac{|1 - 4E-13\omega^2|}{\left|\frac{j\omega}{1.579E6} + 1\right| \left|\frac{j\omega}{1.583E6} + 1\right|}$$

$$\theta(\omega) = 0^\circ - \arctan\left(\frac{\omega}{1.579E6}\right) + 180^\circ - \arctan\left(\frac{\omega}{1.583E6}\right)$$

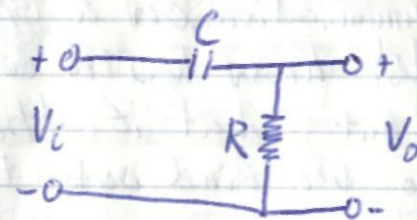
Why add the 180° phase shift? This phase shift occurs when the inductor impedance exceeds the capacitor impedance. At what ω does this occur?

$$\omega L > \frac{1}{\omega C}$$

$$\omega^2 > \frac{1}{LC}$$

$$\omega > \sqrt{\frac{1}{LC}} \triangleq \omega_0$$

ex



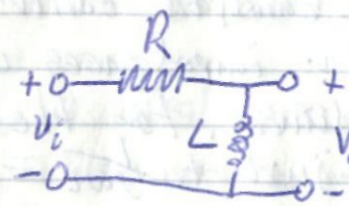
$$\omega_0 = 500 \text{ rad/s}$$

$$C = 1 \text{ mF}$$

$$R = 2 \Omega$$

$$\frac{1}{RC} = \omega_0$$

ex

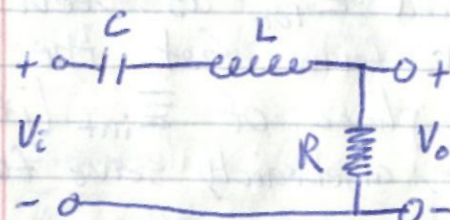


$$\omega_0 = 10 \text{ k rad/s}$$

$$L = 1 \text{ mH}$$

$$R = 10 \Omega$$

$$\frac{R}{L} = \omega_0$$



$$\beta = 9 \text{ kHz} \quad f_1 = 1 \text{ kHz} \quad f_2 = 10 \text{ kHz}$$

$$C = 1 \text{ uF}$$

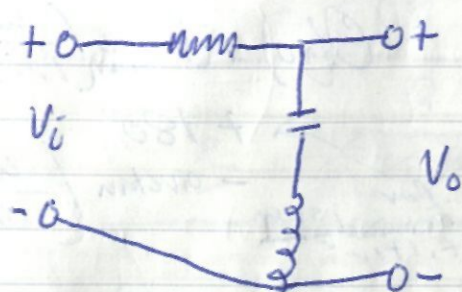
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\sqrt{LC} = \left(\frac{1}{\omega_0}\right)^2$$

$$L = \frac{\left(\frac{1}{\omega_0}\right)^2}{C}$$

And $\rightarrow \omega_0 = 6.32 \pi \text{ krad/s}$
 $L = 2.61 \text{ mH}$
 $C = 1 \text{ uF}$
 $\omega_1 = 2 \pi \text{ krad/s}$
 $\omega_2 = 20 \pi \text{ krad/s}$
 $\beta = 18 \pi \text{ krad/s}$
 $Q = 0.351$
 $K_0 = -7$
 $R = 147.67 \Omega$

ω
rad/s
 f
Hz



$$\beta = 12 \text{ kHz} \quad f_0 = 89.5 \text{ MHz}$$

$$L = 2.2 \text{ mH}$$

$$\omega_0 = 179 \pi \text{ rad/s} \quad K_0 = \text{N/A}$$

$$\omega_1 = 178.938 \pi \text{ rad/s} \quad R = 168.9 \Omega$$

$$\omega_2 = 179.012 \pi \text{ rad/s} \quad L = 2.2 \text{ mH}$$

$$\beta = 24 \pi \text{ krad/sec} \quad C = 1.43 \text{ fF}$$

$$Q = 7458.3$$

(femto)

(Figure out formula)

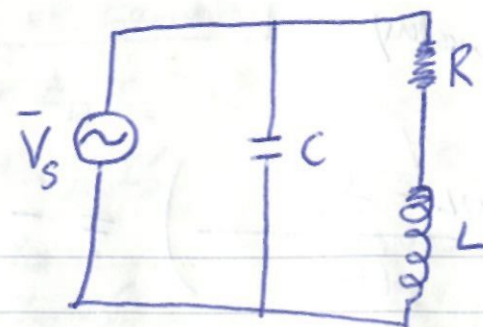
Resonance

Resonance is the AC version of max power transfer but at a specific frequency. Wireless communications systems rely upon resonance by one or two-way communications on a specific channel, i.e. at a specific frequency. We know how to determine ω_0 for the series RLC filter, but what if the filter isn't the series topology? How do we find out the formula or even the value of ω_0 ?

Let us recall the general definition presented briefly at the beginning of the RLC bandpass filter example. ω_0 equals the frequency where all inductor impedances are counterbalanced by all the capacitor impedances. For the series topology filter, that definition told us $j\omega L + \frac{1}{j\omega C} = 0$. Solving for ω yielded $\omega_0 = \sqrt{\frac{1}{LC}}$.

Generally, our process is to find $\bar{Z}_{\text{tot}}$ as seen by the source. Put this quantity in polar form. Set the phase of $\bar{Z}_{\text{tot}}$ equal to the negative of the phase of $\bar{Z}_{\text{int}}$ (if present) or 0° (if $\bar{Z}_{\text{int}}$ is absent). Finally, algebraically solve for ω_0 which will be ω_0 .

↓ example (next page)



$$\omega_0 = ?$$

$\omega_0 \neq \frac{1}{\sqrt{LC}}$ since this isn't the series RLC circuit.
Calculate $\bar{Z}_{\text{tot}}$

$$\bar{Z}_{\text{tot}} = \frac{(R + j\omega L) \left(\frac{1}{j\omega C} \right)}{(R + j\omega L) + \frac{1}{j\omega C}} \quad \text{Simplify to convert to polar form}$$

$$\bar{Z}_{\text{tot}} = \frac{\left(\frac{L}{C} - \frac{jR}{\omega C} \right)}{(R + j[\omega L - \frac{1}{\omega C}])} \quad \text{convert to polar}$$

$$\bar{Z}_{\text{tot}} = \frac{\sqrt{\left(\frac{L}{C} \right)^2 + \left(\frac{-R}{\omega C} \right)^2} \left[\tan^{-1} \left(\frac{(-R/\omega C)}{(\frac{L}{C})} \right) \right]}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} \left[\tan^{-1} \left(\frac{\omega L - \frac{1}{\omega C}}{R} \right) \right]}$$

divide

$$\bar{Z}_{\text{tot}} = \sqrt{\frac{\left(\frac{L}{C} \right)^2 + \left(\frac{-R}{\omega C} \right)^2}{(R)^2 + (\omega L - \frac{1}{\omega C})^2}} \left[\tan^{-1} \left(\frac{-R/\omega C}{\frac{L}{C}} \right) - \tan^{-1} \left(\frac{\omega L - \frac{1}{\omega C}}{R} \right) \right]$$

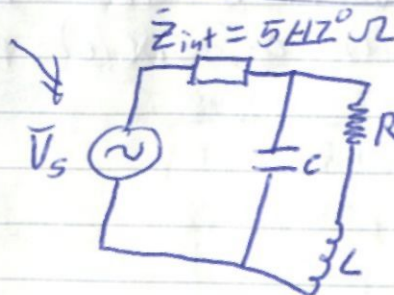
$\bar{V}_s$ is ideal or $\bar{Z}_{\text{int}}$ is absent $\rightarrow$ set the phase $= 0^\circ$
solve for $\omega \rightarrow$ will be ω_0

$$\bar{Z}_{\text{tot}} = \sqrt{\frac{\left(\frac{L}{C} \right)^2 + \left(\frac{-R}{\omega C} \right)^2}{(R)^2 + (\omega L - \frac{1}{\omega C})^2}}$$

do Algebra! to find
 $\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \text{ rad/s}$

Suppose $\bar{V}_s$ had been non ideal.
It would have had $\bar{Z}_{\text{int}}$ eg

Then our work would have shown



Thursday

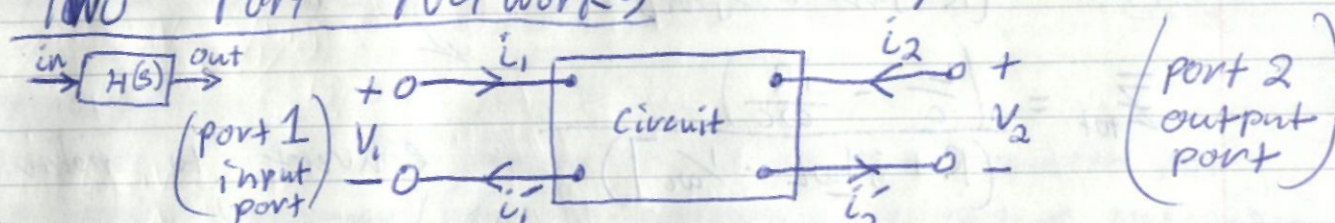
$$\arctan\left(\frac{(-R)}{\left(\frac{L}{C}\right)}\right) - \arctan\left(\frac{(\omega L - \frac{1}{\omega C})}{(R)}\right) = -17^\circ$$

Then solve for ω - which will be ω_0

$$\omega_0 \neq \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \text{ Rad/s}$$

Two Port Networks

Thursday NOV. 6 2025



A pair of terminals is referred to as a port
Use of this diagram requires the following restrictions:

1. no energy is stored in the circuit
2. no independent sources in the circuit
3. $i_1 = i_1'$, $i_2 = i_2'$ (prime notation does not denote derivative)
4. all external connections are made across the ports, not between the ports. (can't connect port 1 and port 2 in any way)

With these restrictions, only V_1, i_1, V_2, i_2 are of interest. 6 kinds of parameters: z, y, a, b, g, h

*: immittance parameters

Δ : transmission parameters

$\square$: hybrid parameters

Only 2 variables are independent $\therefore$, one always has one (1) of six (6) possible systems of equations.

6 Systems of Equations

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$V_1 = a_{11} V_2 - a_{12} I_2$$

$$I_1 = a_{21} V_2 - a_{22} I_2$$

$$V_2 = b_{11} V_1 - b_{12} I_1$$

$$I_2 = b_{21} V_1 - b_{22} I_1$$

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$I_1 = g_{11} V_1 + g_{12} I_2$$

$$V_2 = g_{21} V_1 + g_{22} I_2$$

Consider the Z parameters ...

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

leave Port 2 open i.e. let $i_2 = 0A$

$$Z_{11} = \frac{V_1}{i_1} \bigg|_{i_2=0A} \quad \Omega \quad \left(\text{impedance looking into Port 1 with Port 2 open} \right)$$

$$Z_{21} = \frac{V_2}{i_1} \bigg|_{i_2=0A} \quad \Omega \quad \left(\text{transfer impedance with Port 2 open} \right)$$

Leave Port 2 open $\rightarrow$ let $i_1 = 0A$

$$Z_{12} = \frac{V_1}{i_2} \bigg|_{i_1=0A} \quad \Omega \quad \left(\text{transfer impedance with port 1 open} \right)$$

$$Z_{22} = \frac{V_2}{i_2} \bigg|_{i_1=0A} \quad \Omega \quad \left(\text{impedance looking into Port 2 with port 1 open} \right)$$

Conversions

Algebraically transform a system of equations to resemble the desired system of equations; then compare coefficients.

Consider transforming the Y -parameters to Z param. Solve the equations of the Y -parameter for V_1 & V_2 . The coefficients of i_1 & i_2 are the Z parameters.

example "fun"

Port 2 ~~closed~~ open

Port 2 closed

Determine the

answer
 $h_{11} = 1200 \Omega$ $h_{12} = 5E-5$
 $h_{21} = 50$ $h_{22} = 1.25E-5$
 $V_1 = 10mV$
 $i_1 = 10\mu A$
 $V_2 = -40V$

$V_1 = 24mV$
 $i_1 = 20\mu A$
 $i_2 = 1mA$

h-param.

$$V_1 = a_{11} V_2 - a_{12} I_2$$

$$I_1 = a_{21} V_2 - a_{22} I_2$$

Port 2 closed $\rightarrow V_2 = 0V$
 Port 2 open $\rightarrow I_2 = I_2' = 0A$

$$10mV = h_{11} 10\mu A + h_{12} (-40V)$$

$$0 = h_{21} 10\mu A + h_{22} (-40)$$

must use a-parameters system because we know because V_2 and I_2 are zero

Port 2 open

$$10mV = a_{11}(0) - a_{12}(0)$$

$$10\mu A = a_{21}(0) - a_{22}(0)$$

$$10mV = a_{11}$$

$$10\mu A = a_{21}$$

Port 2 closed

Reciprocal Two Port Networks

Tue. Nov. 11 2020

For a two port network to be reciprocal, its parameters must meet the following conditions:

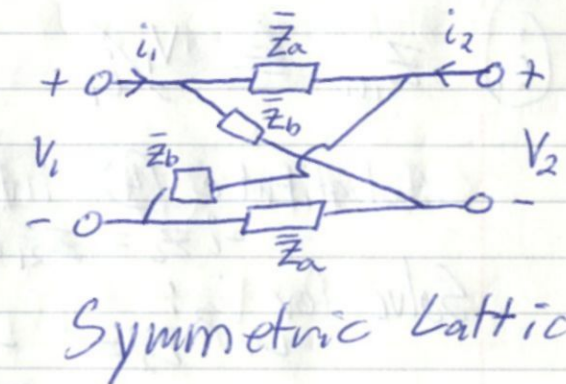
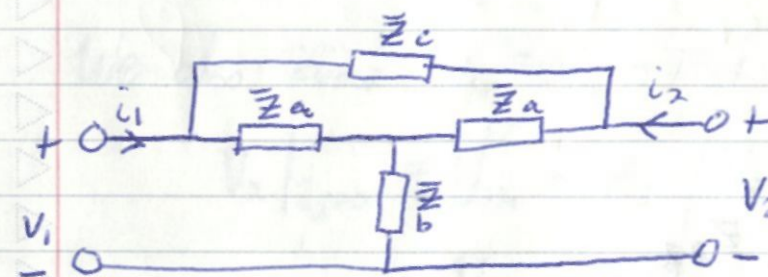
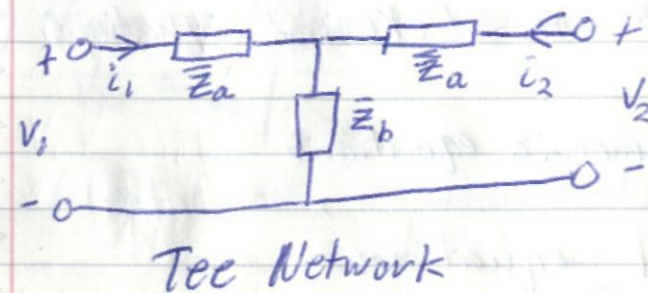
- ① $Z_{12} = Z_{21}$
 - ② $Y_{12} = Y_{21}$
 - ③ $\Delta a = 1$
 - ④ $\Delta b = 1$
 - ⑤ $h_{12} = -h_{21}$
 - ⑥ $g_{12} = -g_{21}$
- Physically this means that one has an ideal voltage (current) source @ one port and an ideal ammeter (voltmeter) at the other port; swapping the ports does not affect the ammeter (voltmeter) reading

Added to this is, a two-port network may be symmetric if the additional conditions below are met:

1. $Z_{11} = Z_{22}$
2. $Y_{11} = Y_{22}$
3. $a_{11} = a_{22}$
4. $b_{11} = b_{22}$
5. $\Delta h = 1$
6. $\Delta g = 1$

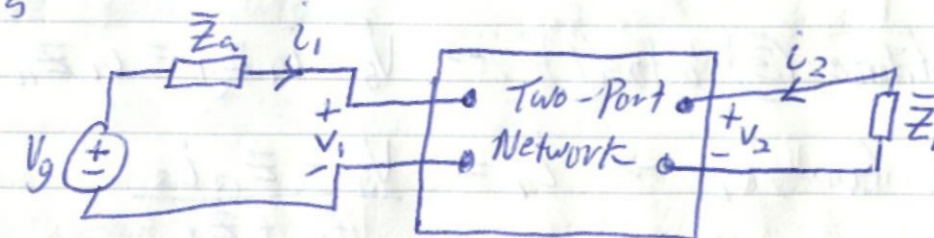
if a circuit has symmetry it is reciprocal

There are 4 common types of two-port networks.



Terminated two Port networks

thus far, one has encountered two-port networks without anything connected to them i.e. unterminated two-port networks. Now we shall have terminated two-port networks



To define its terminal behavior, we have six (6) characteristics

1. input impedance: $\bar{Z}_{in} = \frac{V_1}{I_1}$
 $\bar{Y}_{in} = \frac{I_1}{V_1}$ Alternatively

2. i_2

3. Thevenin voltage & impedance w.r.t. Port 2

4. current gain i_2/i_1 5. voltage gain V_2/V_1 6. Voltage Gain V_2/V_g

Derivation of these six(6) characteristics means using a set of parametric equations & the terminal equations.

$$\begin{aligned} \textcircled{1} V_1 &= i_1 Z_{11} + i_2 Z_{12} \\ \textcircled{2} V_2 &= i_1 Z_{21} + i_2 Z_{22} \end{aligned} \quad \left. \begin{array}{l} \textcircled{1} \\ \textcircled{2} \end{array} \right\} \text{parametric equations}$$

$$\begin{aligned} \textcircled{3} V_1 &= V_g - i_1 Z_g \\ \textcircled{4} V_2 &= -i_2 Z_L \end{aligned} \quad \left. \begin{array}{l} \textcircled{3} \\ \textcircled{4} \end{array} \right\} \text{terminal equations}$$

$$\textcircled{i} Z_{in} \triangleq V_1 / i_1$$

Substitute $\textcircled{4}$ into $\textcircled{2}$

$$-i_2 Z_L = Z_{21} i_1 + i_2 Z_{22}$$

Solve for i_2 :

$$\rightarrow i_2 = \frac{-Z_{21} i_1}{Z_L + Z_{22}} \quad \leftarrow \textcircled{ii}$$

Substitute $\textcircled{ii}$ into $\textcircled{1}$ & solve for V_1 / i_1

$$Z_{in} = Z_{11} - \frac{Z_{21} Z_{12}}{Z_L + Z_{22}}$$

$\textcircled{iii}$

Substitute $\textcircled{3}$ into $\textcircled{1} \rightarrow V_g - i_1 Z_g = i_1 Z_{11} + i_2 Z_{12}$

$$\text{Solve for } i_1 \rightarrow i_1 = \frac{V_g - Z_{12} i_2}{Z_{11} + Z_g}$$

$$\text{Substitute this into } \textcircled{ii} \rightarrow i_2 = \frac{-Z_{21} \left(\frac{V_g - Z_{12} i_2}{Z_{11} + Z_g} \right)}{Z_L + Z_{22}}$$

Solve for i_2

$$i_2 = \frac{-Z_{21} V_g}{[(Z_L + Z_{22})(Z_{11} + Z_g) - Z_{21} Z_{12}]}$$

$\textcircled{iii}$ Thevenin Voltage w.r.t. Port 2

$$V_{th} = V_2 |_{i_2=0}$$

With $i_2=0$, combine $\textcircled{1}$ & $\textcircled{2}$

$$Z_{21} i_1 \rightarrow Z_{21} \left(\frac{V_1}{Z_{11}} \right) = V_2 |_{i_2=0}$$

We also know $V_1 = V_g - Z_g i_1$ & $i_1 = \frac{V_g}{Z_g + Z_{11}}$

$$V_2 |_{i_2=0} = V_{th} = Z_{21} \left(\frac{V_g - \left(\frac{V_g}{Z_g + Z_{11}} \right) Z_g}{Z_{11}} \right)$$

$$V_{th} = \frac{Z_{21} V_g}{Z_g + Z_{11}}$$

Thevenin impedance w.r.t. Port 2

$$Z_{th} \triangleq \frac{V_2}{i_2} |_{V_g=0}$$

With $V_g=0V$, $\textcircled{3}$ becomes $V_1 = -i_1 Z_g$

Substitute this into $\textcircled{1}$

$$-i_1 Z_g = Z_{11} i_1 + Z_{12} i_2$$

Solve for i_1

$$i_1 = \frac{-Z_{12} i_2}{Z_{11} + Z_g}$$

Substitute this into $\textcircled{2}$ & solve for V_2 / i_2

$$Z_{th} = Z_{22} - \frac{Z_{12} Z_{21}}{Z_{11} + Z_g}$$

(iv) Current gain $i_2/i_1 \rightarrow$ solve (i) for i_2/i_1

(v) Voltage Gain V_2/V_1

Solve for i_2 in (4) $\rightarrow i_2 = -V_2/Z_L$

Solve for i_1 in (1) $\rightarrow i_1 = \frac{V_1 - Z_{12} i_2}{Z_{11}}$

Substitute (4) into (2)

$$\rightarrow V_2 = Z_{21} i_1 + Z_{22} \left(-\frac{V_2}{Z_L} \right)$$

Substitute (4) into (1)

$$\rightarrow i_1 = \frac{V_1 + Z_{12} (V_2/Z_L)}{Z_{11}}$$

Substitute (4) into (2) & solve for V_2/V_1

$$\rightarrow \frac{V_2}{V_1} = \frac{Z_{21} Z_L}{Z_{11} Z_L + \Delta Z}$$

(vi) Voltage gain V_2/V_g

Substitute (3) & (4) into (1) $\rightarrow V_g - i_1 Z_g = Z_{11} i_1 + Z_{12} \left(-\frac{V_2}{Z_L} \right)$

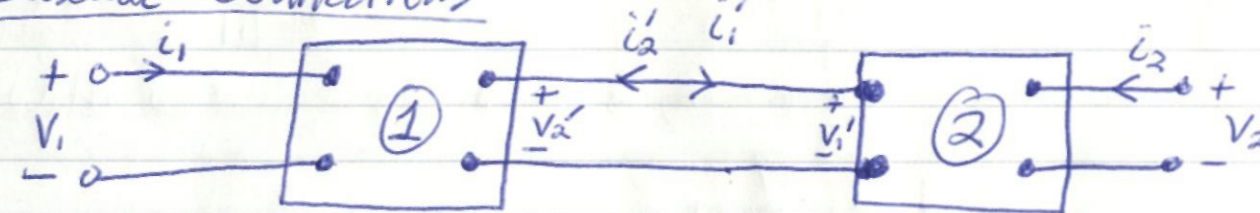
$$\text{Solve for } i_1 \rightarrow i_1 = \frac{V_g + \frac{Z_{12}}{Z_L} V_2}{Z_{11} + Z_g}$$

Substitute (6) & (4) into (2)

$$V_2 = Z_{21} \left[\frac{V_g + \frac{Z_{12}}{Z_L} V_2}{Z_{11} + Z_g} \right] + Z_{22} \left(-\frac{V_2}{Z_L} \right)$$

$$\text{Solve for } V_2/V_g \rightarrow \frac{V_2}{V_g} = \frac{Z_{21} Z_L}{(Z_{11} + Z_g)(Z_{22} + Z_L) - Z_{12} Z_{21}}$$

Cascade Connections



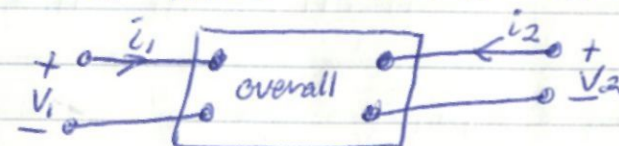
$$\text{Overall } \begin{cases} V_1 = a_{11} V_2 - a_{12} i_2 \\ i_1 = a_{21} V_2 - a_{22} i_2 \end{cases}$$

$$\text{(1) } \begin{cases} V_1 = a_{11} V_2' - a_{12} i_2' \\ i_1 = a_{21} V_2' - a_{22} i_2' \end{cases} \rightarrow \begin{cases} V_1 = a_{11} V_1' + a_{12} i_1' \\ i_1 = a_{21} V_1' + a_{22} i_1' \end{cases}$$

$$\text{(2) } \begin{cases} V_1' = a_{11}' V_2 - a_{12}' i_2 \\ i_1' = a_{21}' V_2 - a_{22}' i_2 \end{cases} \text{ substitute}$$

$$\begin{aligned} V_1 &= (a_{11} a_{11}' + a_{12} a_{21}') V_2 - (a_{11} a_{12}' + a_{12} a_{22}') i_2 \\ i_1 &= (a_{21} a_{11}' + a_{22} a_{21}') V_2 - (a_{21} a_{12}' + a_{22} a_{22}') i_2 \end{aligned}$$

as without prime is from network (1)
as with prime notation is from network (2)



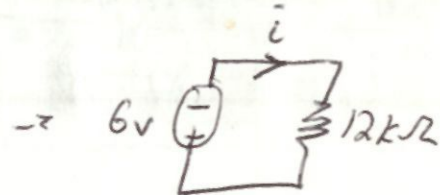
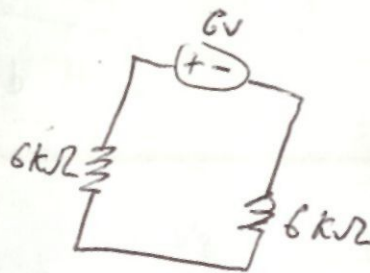
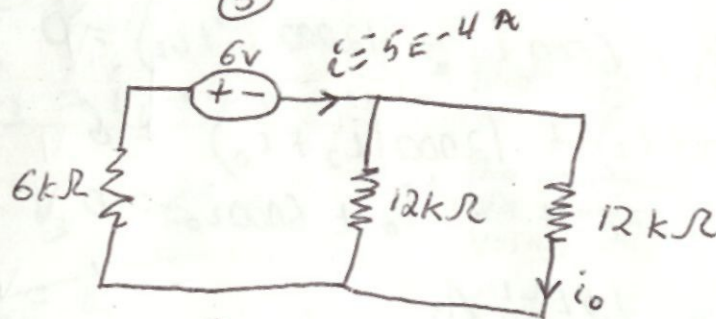
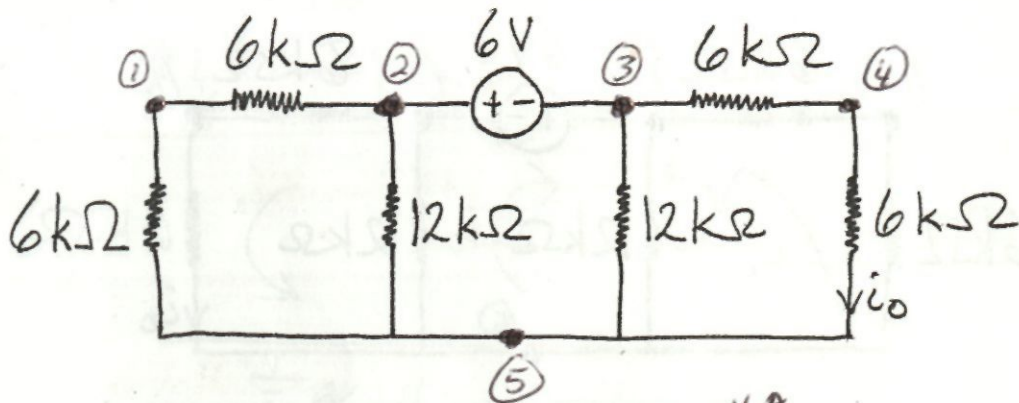
You would apply recursively

FIN

Circuits II

Blank Examples

Reductions

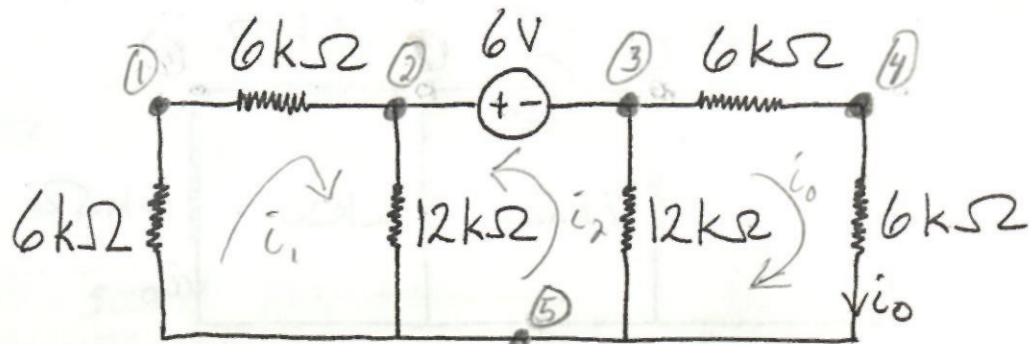


$$V = iR$$

$$i = \frac{6}{12 \times 10^3} = 5 \times 10^{-4} \text{ A}$$

$$i_o = \frac{-(5 \times 10^{-4})(12 \times 10^3)}{(12 \times 10^3) + (12 \times 10^3)} = -2.5 \times 10^{-4} \text{ A}$$

KVL



$$6000 i_1 + 6000 i_1 + 12000 (i_1 + i_2) = 0$$

$$12000 (i_2 + i_1) + 12000 (i_2 + i_0) = 6$$

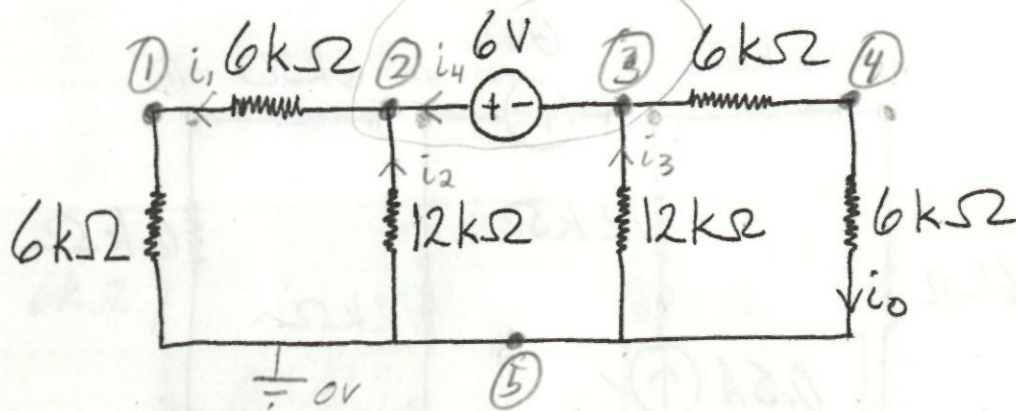
$$12000 (i_0 + i_2) + 6000 i_0 + 6000 i_0 = 0$$

$$i_1 = -2.5E-4 A$$

$$i_2 = 5E-4 A$$

$$i_0 = -2.5E-4 A$$

KCL



$$V = IR$$

$$I = \frac{V}{R}$$

(SN)

$$i_2 + i_3 = i_1 + i_0$$

(SpE)

$$V_2 - V_3 = 6$$

~~(2)~~

~~$$i_4 + i_2 = i_1$$~~

~~(3)~~

~~$$i_3 = i_0 + i_4$$~~

- Node ① does not need a KCL Equation because the current going in is the same as the current going out.

- Node ⑤ does not need equation bc its ground
- Nodes ② and ③ do not need equations bc part of supernode.

(SN)

$$\frac{0 - V_2}{12000} + \frac{-V_3}{12000} = \frac{V_2 - V_1}{12000} + \frac{V_3}{12000}$$

(SpE)

$$V_2 - V_3 = 6$$

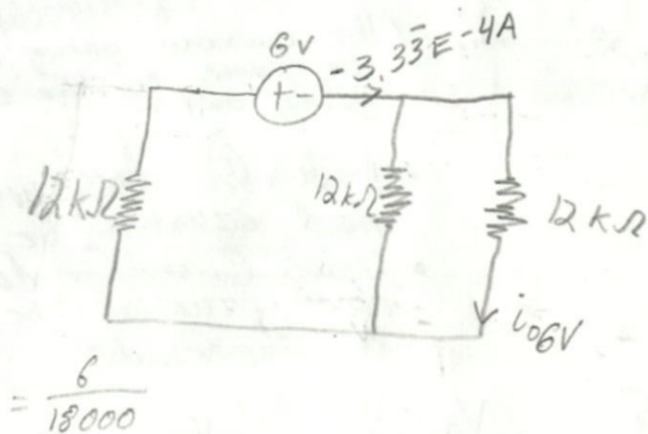
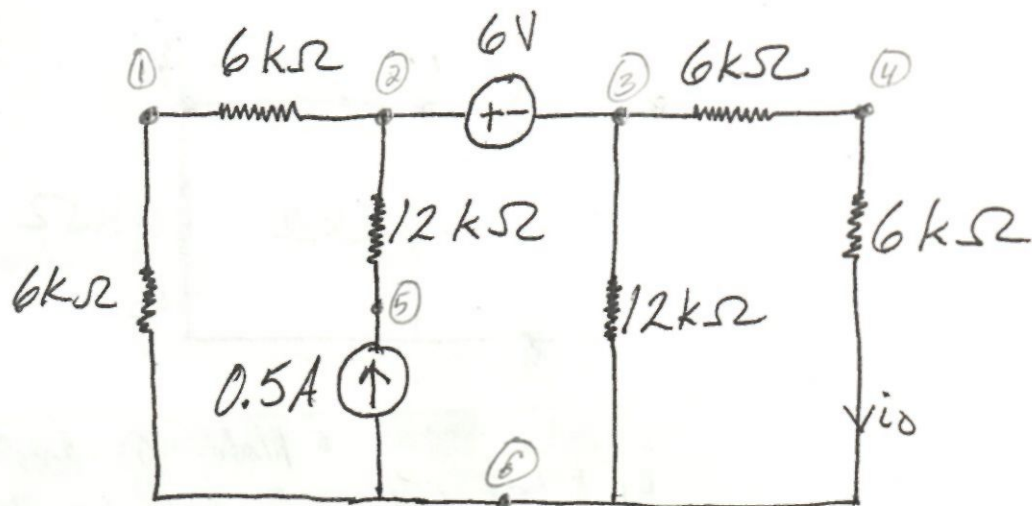
$$V_2 = 3V$$

$$V_3 = -3V$$

$$i_0 = \frac{V_3}{12000} = \frac{-3}{12000}$$

$$i_0 = -2.5 \times 10^{-4} A$$

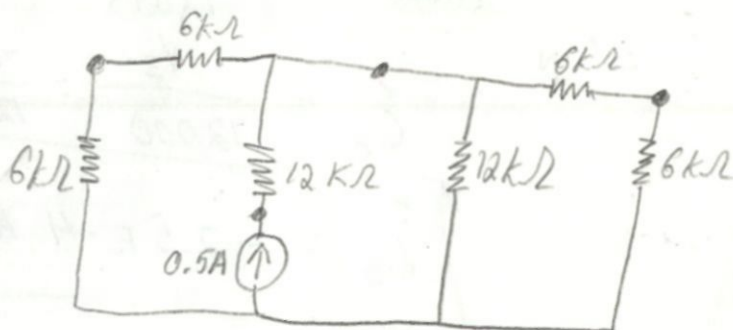
Superposition



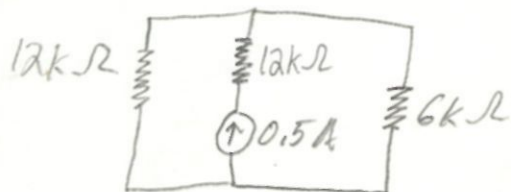
$$i_o = i_{o6V} + i_{o0.5A}$$

$$i_{o6V} = \frac{-(3.33 \times 10^{-4}) (12 \times 10^3)}{(12 \times 10^3) + (12 \times 10^3)}$$

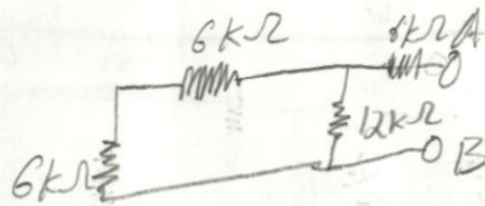
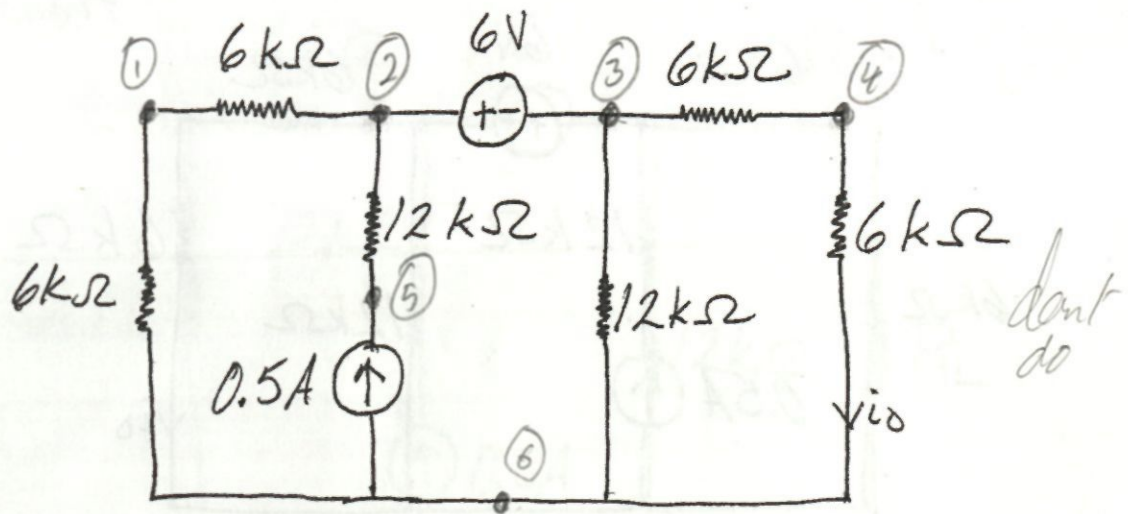
$$i_{o6V} = -1.66 \times 10^{-4} \text{ A}$$



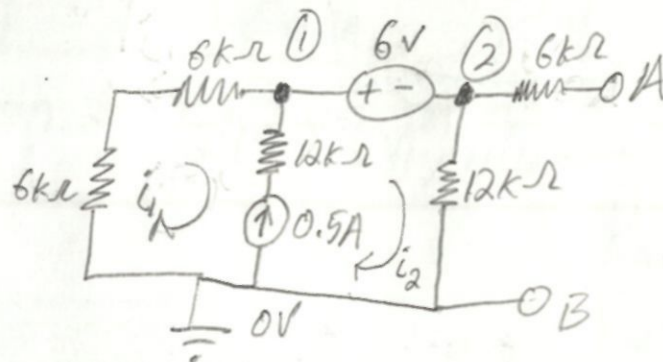
↓



Thevenin Eq Cir



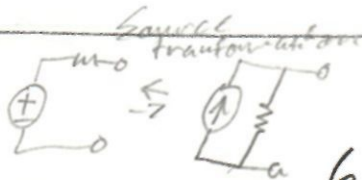
$$R_{th} = 12k\Omega$$



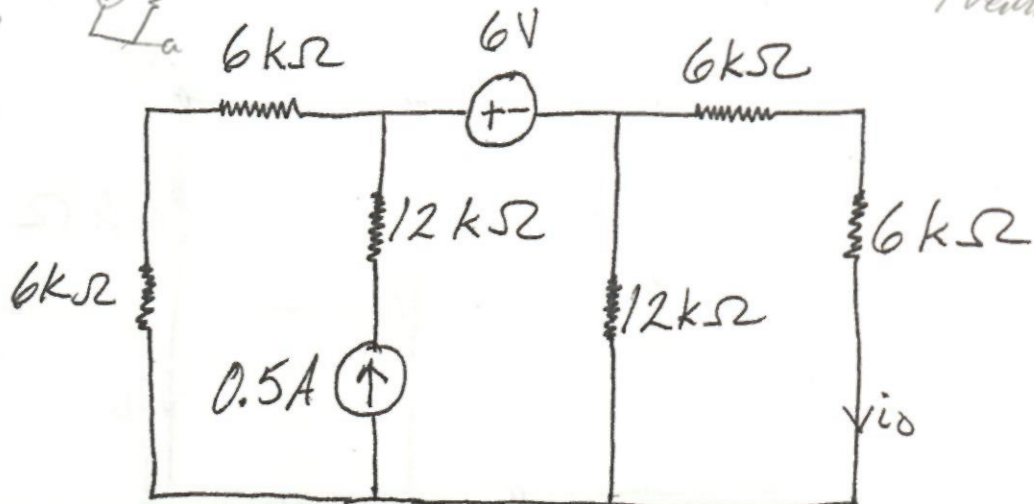
$$i_o = 0.1665A$$

$$12000i_1 + 12000i_2 = -6$$

Norton Eq Cir



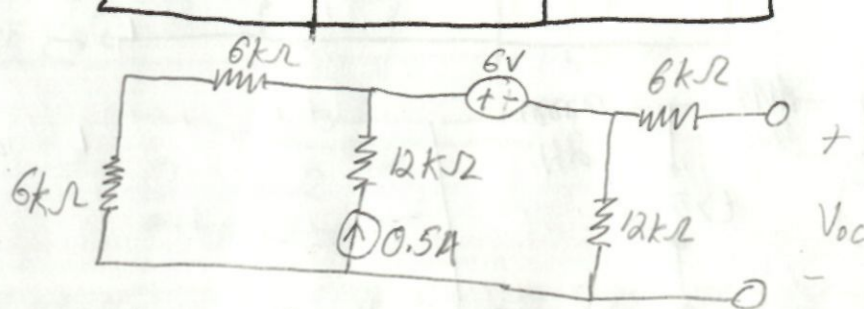
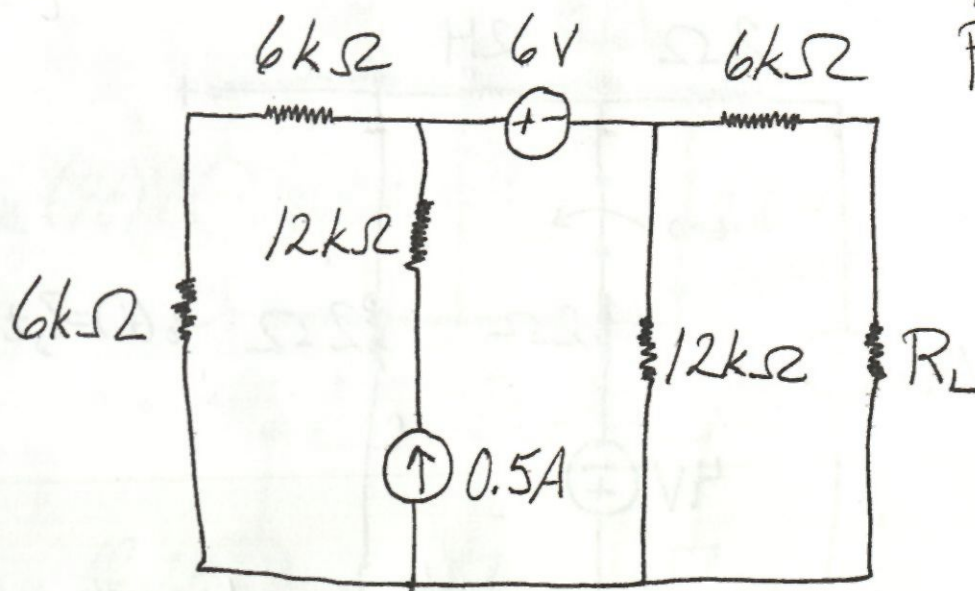
Use Source transformation



Max Power Transfer Review Example

$$R_L = ?$$

$$P_L = ?$$

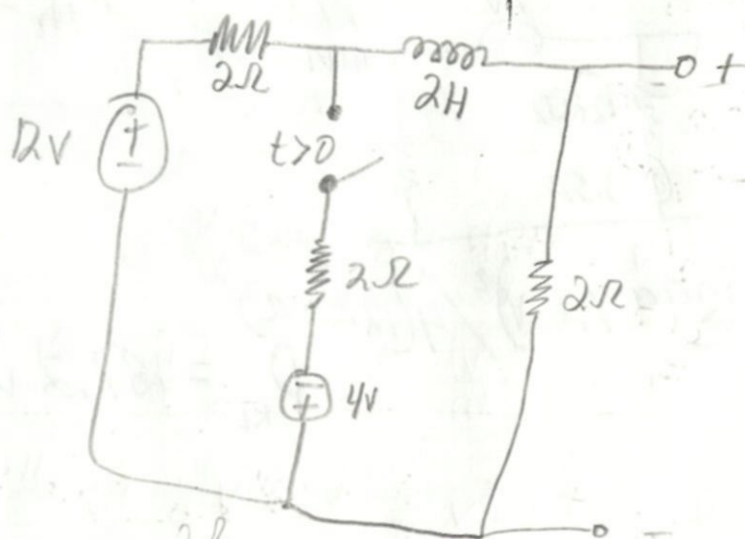
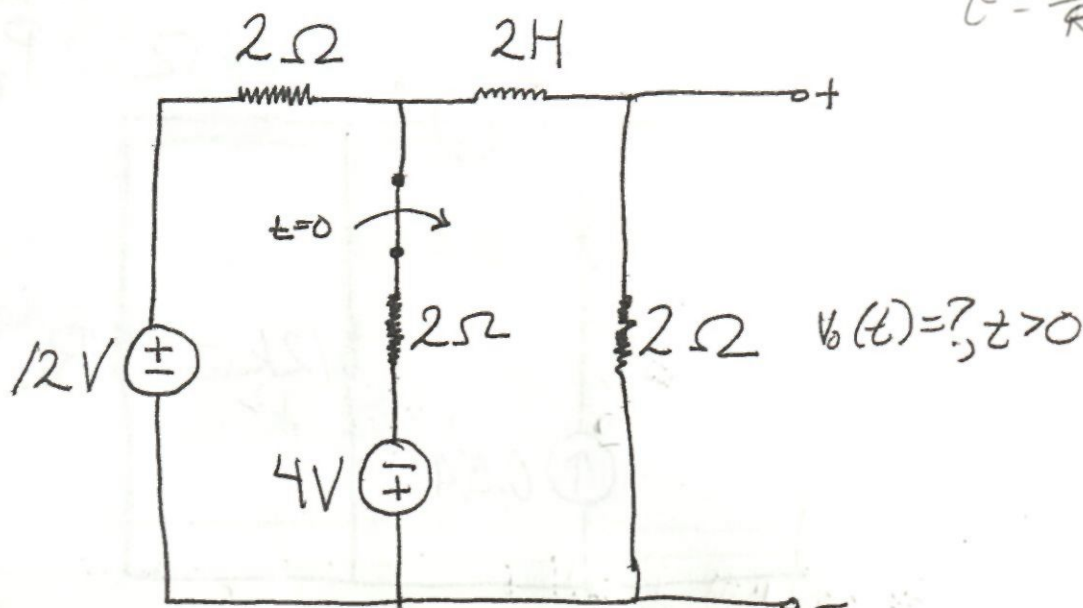


$$R_{th} = 12k\Omega$$

$$P_{R_L} = (2997.6V)^2 / 4(12000\Omega)$$

$$P_{R_L} = 187.2W$$

$$\tau = \frac{L}{R}$$



$$4. V_o(t) = K_1 e^{-\frac{t}{\tau}} + K_2$$

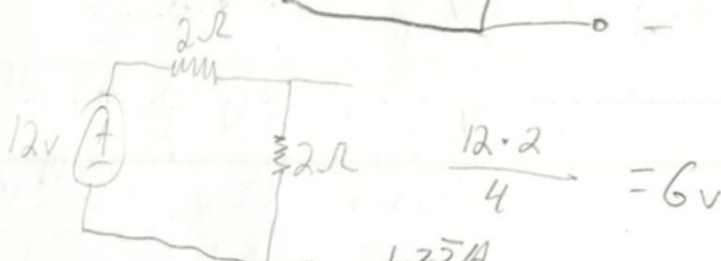
$$V_o(\infty) = 6V$$

by inspection

$$5. K_1 = IC - K_2$$

$$= -3.333 V$$

$$6. V_o(t) = -3.33e^{-2t} + 6 V$$



$$7. \tau = \frac{L}{R_{th}}$$



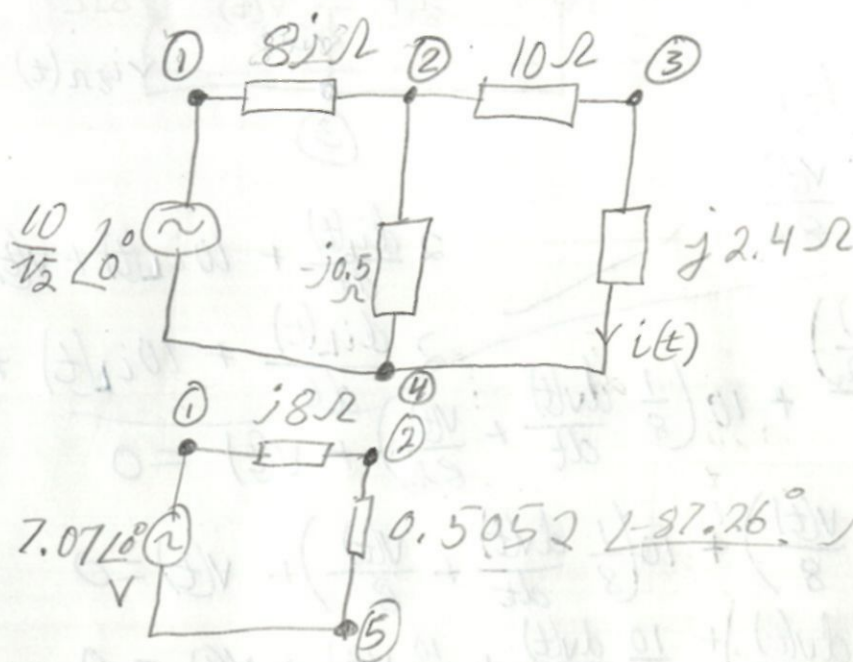
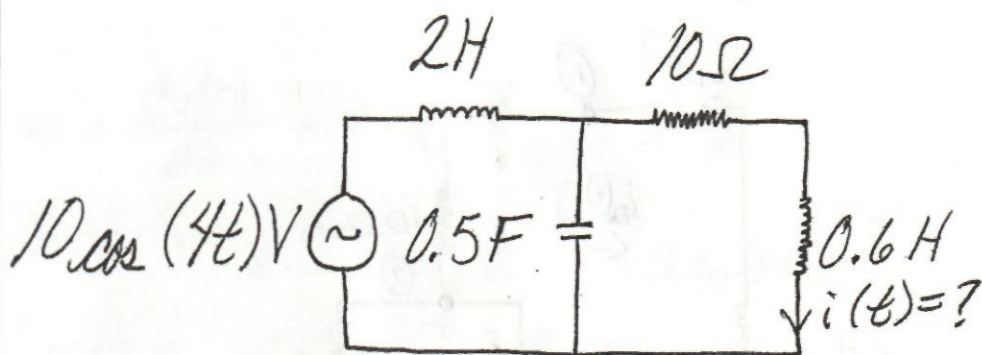
$$R_{th} = 4\Omega \Rightarrow \tau = \frac{2H}{4\Omega}$$

$$\tau = 0.5s$$

AC Analysis

Kirchhoff Example

$$\omega = 4$$



$$\bar{I} = \frac{\bar{V}_{25}}{10 + j2.4 \Omega}$$

$$i(t) = 65.54 \cos(4t + 169.42^\circ)$$

$$P = (4V) \frac{P}{P} + (10V) \frac{P}{P} + (10V) \frac{P}{P} \text{ mW}$$

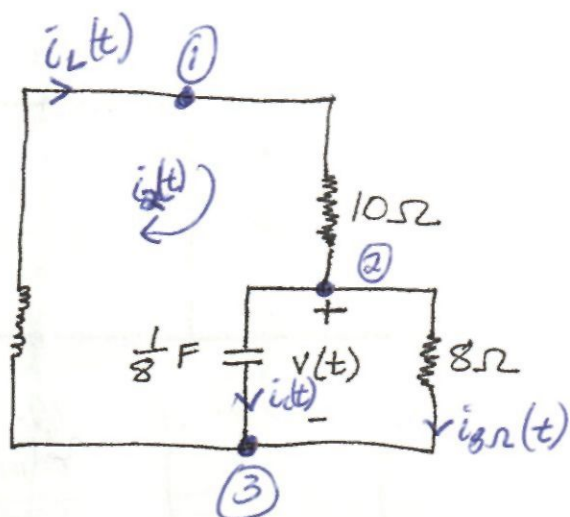
Mixture topology

$$v(t) = ?$$

$$v_c(0) = 1V$$

$$i_L(0) = 0.5A$$

2H



$$i_L(t) = i_c(t) + i_{8\Omega}(t)$$

$$i_L(t) = \frac{1}{8} \frac{dv(t)}{dt} + \frac{v(t)}{8\Omega}$$

$$2 \frac{di_L(t)}{dt} + 10i_L(t) + v(t) = 0$$

$$2 \frac{di_L(t)}{dt} + 10i_L(t) + v(t) = 0$$

$$2 \frac{d}{dt} \left(\frac{1}{8} \frac{dv(t)}{dt} + \frac{v(t)}{8\Omega} \right) + 10 \left(\frac{1}{8} \frac{dv(t)}{dt} + \frac{v(t)}{8\Omega} \right) + v(t) = 0$$

$$2 \left(\frac{1}{8} \frac{d^2v(t)}{dt^2} + \frac{1}{8} \frac{dv(t)}{dt} \right) + \frac{10}{8} \frac{dv(t)}{dt} + \frac{10}{8} v(t) + v(t) = 0$$

$$\frac{1}{4} v''(t) + \frac{1}{4} v'(t) + \frac{5}{4} v'(t) + \frac{10}{8} v(t) + v(t) = 0$$

$$\frac{1}{4} v''(t) + \frac{6}{4} v'(t) + \frac{9}{4} v(t) = 0$$

$$v''(t) + 6v'(t) + 9v(t) = 0$$

$$s^2 + 6s + 9 = 0 \Rightarrow s = -3, -3 \Rightarrow \text{critically damped}$$

$$v(t) = \alpha_1 e^{-3t} + \alpha_2 t e^{-3t} + v(\infty)$$

$$v(\infty) = 0 \text{ by inspection}$$

$$v(t) = \alpha_1 e^{-3t} + \alpha_2 t e^{-3t}$$

$$1 = \alpha_1 e^{-3(0)} + \alpha_2(0) e^{-3(0)} \Rightarrow \boxed{\alpha_1 = 1}$$

$$v(t) = e^{-3t} + \alpha_2 t e^{-3t}$$

$$\frac{dv(t)}{dt} = -3e^{-3t} + \alpha_2 e^{-3t} - 3\alpha_2 t e^{-3t}$$

$$i_L(0) = \frac{1}{8} \frac{dv(0)}{dt} + \frac{v(0)}{8}$$

$$[0.5A = \frac{1}{8} \frac{dv(0)}{dt} + \frac{1}{8}] 8$$

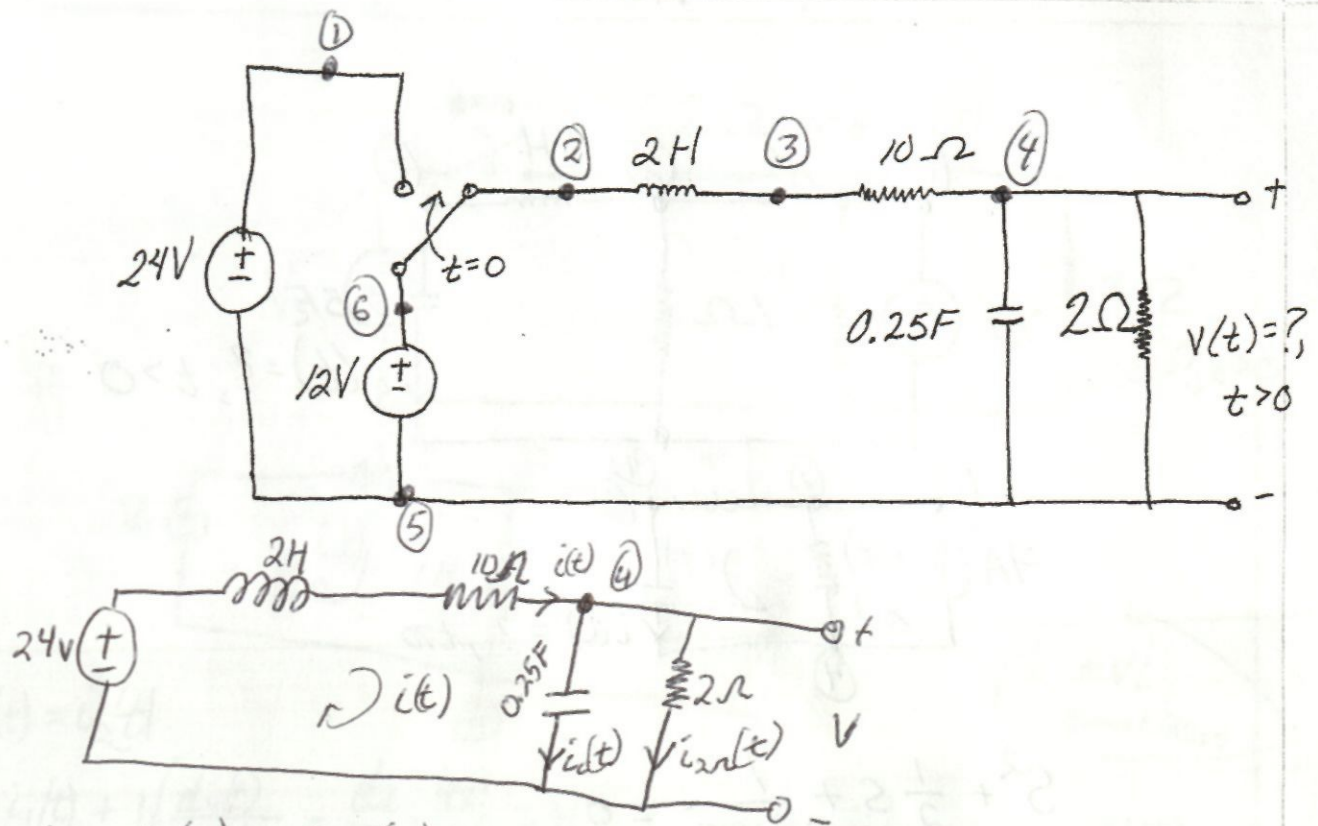
mixture topology cont.

$$4 = \frac{dv(0)}{dt} + 1 \Rightarrow \frac{dv(0)}{dt} = 3$$

$$3 = -3e^{-3(0)} + \alpha_2 e^{-3(0)} - 3\alpha_2(0)e^{-3(0)} \Rightarrow \alpha_2 = 6$$

$$\underline{v(t) = e^{-3t} + 6te^{-3t} \quad V, \quad t \geq 0}$$

mixture



$$i(t) = i_c(t) + i_{2\Omega}(t)$$

$$\frac{v(t)}{2} + 0.25 \frac{dv(t)}{dt} = i(t) \quad 2 \frac{di(t)}{dt} + 10i(t) + v(t) = 24$$

KCL $\rightarrow$ KVL
Substitute

$$2 \frac{d}{dt} \left(\frac{v(t)}{2} + 0.25 \frac{dv(t)}{dt} \right) + 10 \left(\frac{v(t)}{2} + 0.25 \frac{dv(t)}{dt} \right) + v(t) = 24$$

$$V' + \frac{1}{2} V'' + 5V + 2.5V' + V = 24$$

$$\frac{1}{2} V'' + \frac{7}{2} V' + 6V = 24 \rightarrow V'' + 7V' + 12V = 48$$

$$S^2 + 7S + 12 = 0 \Rightarrow S = -3, -4 \text{ overdamped}$$

$$V(t) = \alpha_1 e^{s_1 t} + \alpha_2 e^{s_2 t} + V(\infty) ; V(\infty) = 4V$$

$$x(t) = \alpha_1 e$$

$$2V = \alpha_1 e^{-3(0)} + \alpha_2 e^{-4(0)} + 4 \Rightarrow \alpha_1 + \alpha_2 = -2$$

$$V' = -3\alpha_1 e^{-3t} - 4\alpha_2 e^{-4t}$$

$$\frac{V(0)}{2} + 0.25 \frac{dV(0)}{dt} = i(0) \text{ and } i(0) = 1A$$

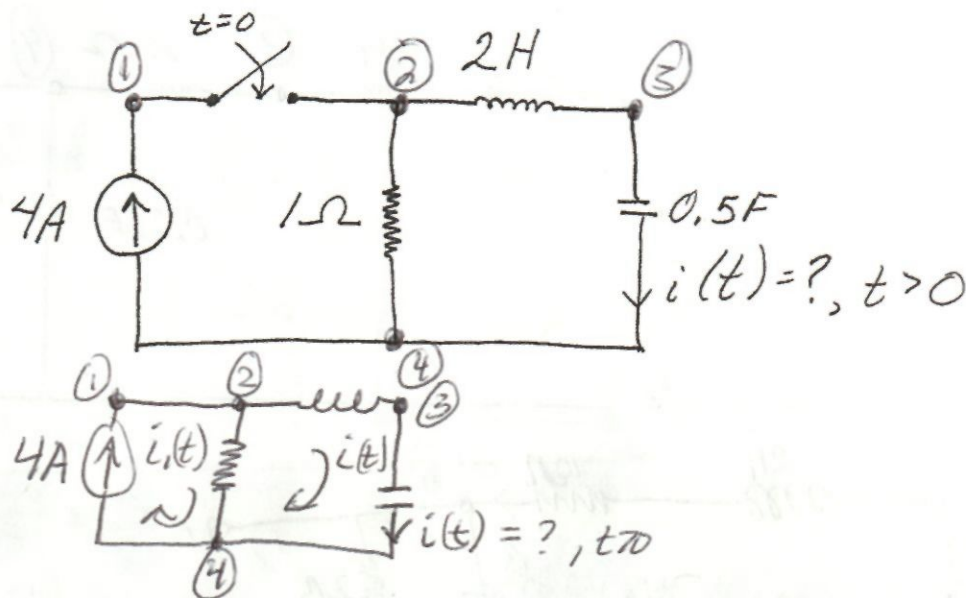
$$\frac{2V}{2} + 0.25 \frac{dV(0)}{dt} = 1A \Rightarrow \frac{dV(0)}{dt} = 0 \frac{V}{s}$$

$$0 = -3\alpha_1 e^{-3(0)} - 4\alpha_2 e^{-4(0)} \Rightarrow -3\alpha_1 - 4\alpha_2 = 0$$

$$\alpha_1 = -8 \quad \alpha_2 = 6$$

$$V(t) = -8e^{-3t} + 6e^{-4t} + 4V \quad t > 0$$

mixture



$$s^2 + \frac{1}{2}s + \frac{1}{2 \cdot 0.5} = 0$$

$$s^2 + \frac{1}{2}s + 1 = 0 \quad \text{underdamped} \quad s = -0.25 \pm j0.9682$$

$$i(t) = \alpha_1 e^{(-0.25 + j0.9682)t} + \alpha_2 e^{(-0.25 - j0.9682)t} + i(\infty) \quad i(\infty) = 0 \text{ inspection}$$

$$i(0) = \alpha_1 e^{(-0.25 + j0.9682)(0)} + \alpha_2 e^{(-0.25 - j0.9682)(0)} \quad i(0) = 0$$

$$= \alpha_1 + \alpha_2$$

$$\frac{di(t)}{dt} = (-0.25 + j0.9682)\alpha_1 e^{(-0.25 + j0.9682)t} + (-0.25 - j0.9682)\alpha_2 e^{(-0.25 - j0.9682)t}$$

$$1(i(0) - 4) + 2 \frac{di(0)}{dt} + v(0) = 0$$

$$-4 + 2 \frac{di(0)}{dt} = 0 \Rightarrow \frac{di(0)}{dt} = 2 \text{ A/s}$$

$$2 = (-0.25 + j0.9682)\alpha_1 e^{(-0.25 + j0.9682)(0)} + \dots$$

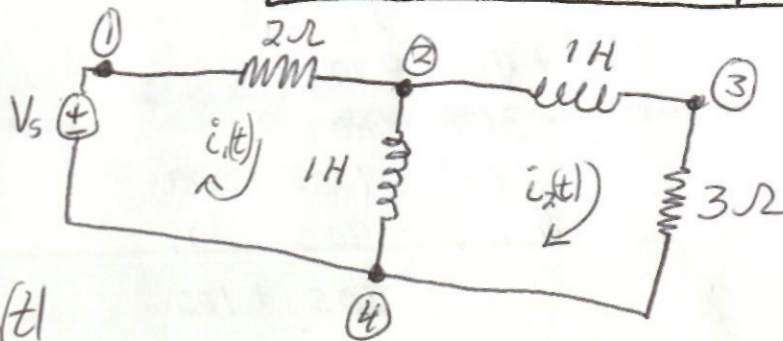
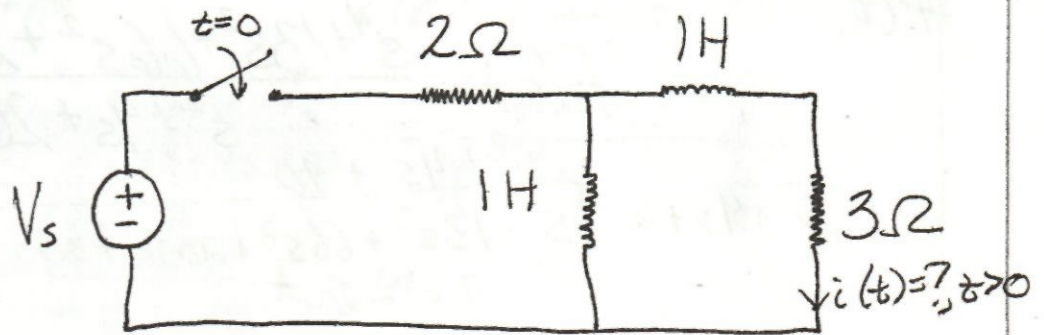
$$2 = j1.9364\alpha_1 \Rightarrow \alpha_2 = -j1.033 \Rightarrow \alpha_2 = j1.033$$

$$i(t) = -j1.033 e^{(-0.25 + j0.9682)t} + j1.033 e^{(-0.25 - j0.9682)t}$$

must simplify (find pattern)

$$\rightarrow i(t) = 2.066 e^{-0.25t} \sin(0.9682t) \text{ A, } t > 0$$

another mixture



2 KVL equations

$$i(t) = i_2(t)$$

$$2i_1(t) + 1\left(\frac{di_1(t)}{dt} - \frac{di_2(t)}{dt}\right) = V_s$$

$$2\frac{di_2(t)}{dt} + 3i_2 - \frac{di_1(t)}{dt} = 0$$

neither KVL substitution
will give you an s^2 term $\rightarrow$ (sad face)

so must use Cramer's Rule

~~$$\begin{aligned} s+2\bar{i}_1 - s\bar{i}_2 &= V_s \\ -s\bar{i}_1 + (2s+3)\bar{i}_2 &= 0 \end{aligned}$$~~

$$\begin{aligned} s+2\bar{i}_1 - s\bar{i}_2 &= V_s \\ -s\bar{i}_1 + (2s+3)\bar{i}_2 &= 0 \end{aligned}$$

By Cramer's Rule: $\bar{i}_2 = \frac{sV_s}{s^2 + 7s + 6} \Rightarrow s^2\bar{i}_2 + 7s\bar{i}_2 + 6\bar{i}_2 = sV_s$

$$i_2'' + 7i_2' + 6i_2 = V_s'$$

$$s^2 + 7s + 6 = 0$$

$$f(t) = ?$$

$$F(s) = \frac{s^4 + 13s^3 + 66s^2 + 200s + 300}{s^2 + 9s + 20}$$

$$\begin{array}{r} s^2 + 9s + 20 \overline{) s^4 + 13s^3 + 66s^2 + 200s + 300} \\ \underline{-s^4 - 9s^3 - 20s^2} \end{array}$$

$$\begin{array}{r} 4s^3 + 46s^2 + 200s \\ \underline{-4s^3 - 36s^2 - 80s} \\ 10s^2 + 120s + 300 \\ \underline{-10s^2 - 90s - 200} \\ 30s + 100 \end{array}$$

$$F(s) = s^2 + 4s + 10 + \frac{30s + 100}{s^2 + 9s + 20}$$

$$\frac{30s + 100}{(s+4)(s+5)} = \frac{A}{(s+4)} + \frac{B}{(s+5)}$$

$$30s + 100 = A(s+5) + B(s+4)$$

$$= As + A5 + Bs + B4$$

$$30s + 100 = s(A+B) + A5 + B4$$

$$\begin{array}{l} A = -20 \\ B = 50 \end{array}$$

$$F(s) = s^2 + 4s + 10 + \frac{-20}{(s+4)} + \frac{50}{(s+5)}$$

$$f(t) = d''(t) + 4d'(t) + 10d(t) - 20e^{-4t} + 50e^{-5t}$$

$$I(s) = \frac{4(s+2)}{s^2+2s+10}$$

$$i(t) = 4.22 e^{-t} \cos(3t + 18.4^\circ) A$$

$$\text{IVT: } s \cdot I(s) = \frac{(s)(4)(s+2)}{s^2+2s+10} = \frac{(s)(4)(s)(1+\frac{2}{s})}{(s^2)(1+\frac{2}{s}+\frac{10}{s^2})}$$

$$\lim_{s \rightarrow \infty} \frac{4(1+\frac{2}{s})}{1+\frac{2}{s}+\frac{10}{s^2}} = 4$$

$$\lim_{t \rightarrow 0^+} 4.22 e^{-t} \cos(3t + 18.4^\circ) = 4$$

FVT:

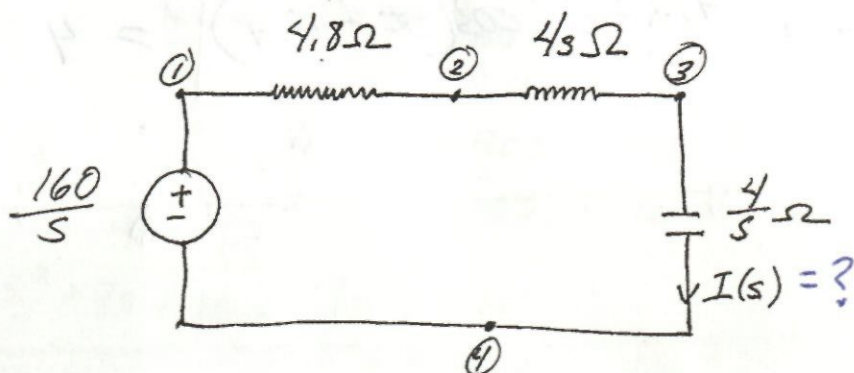
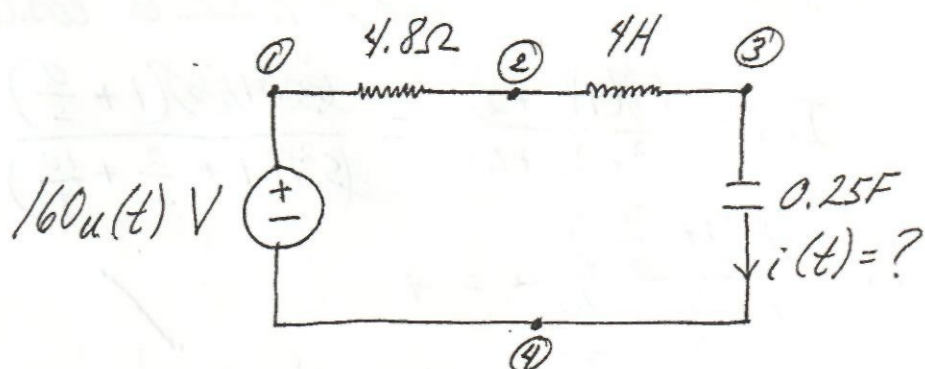
$$s \cdot I(s) = \frac{(s)(4)(s+2)}{s^2+2s+10} = \frac{(s)(4)(s)(1+\frac{2}{s})}{(s^2)(1+\frac{2}{s}+\frac{10}{s^2})}$$

$$\lim_{s \rightarrow 0} \frac{4(1+\frac{2}{s})}{s^2(1+\frac{2}{s}+\frac{10}{s^2})} = 0$$

$$\lim_{t \rightarrow \infty} 4.22 e^{-t} \cos(3t + 18.4^\circ) = 0$$

$$i(0) = 0A$$

$$V(0) = 0V$$



Use Reduction

$$I(s) = \frac{V}{Z}$$

$$I(s) = \frac{\left(\frac{160}{s}\right)}{\left(4s + 4.8 + \frac{4}{s}\right)} \cdot \frac{s}{s}$$

$$I(s) = \frac{(160)}{(4s^2 + 4.8s + 4)} = \frac{(40)}{(s^2 + 1.2s + 1)} = \frac{A}{(s+0.6)} + \frac{B}{(s+0.6-j0.8)}$$

$$\frac{40}{s^2 + 1.2s + 1} = \frac{A}{(s+0.6+j0.8)} + \frac{B}{(s+0.6-j0.8)}$$

$$40 = A(s+0.6-j0.8) + B(s+0.6+j0.8)$$

$$40 = B(-0.6+j0.8+0.6+j0.8) \quad 40 =$$

$$40 = Bj1.6$$

$$B = \frac{40}{j1.6} = -j25$$

$$A = j25$$

$$I(s) = \frac{j25}{(s+0.6+j0.8)} - \frac{j25}{(s+0.6-j0.8)}$$

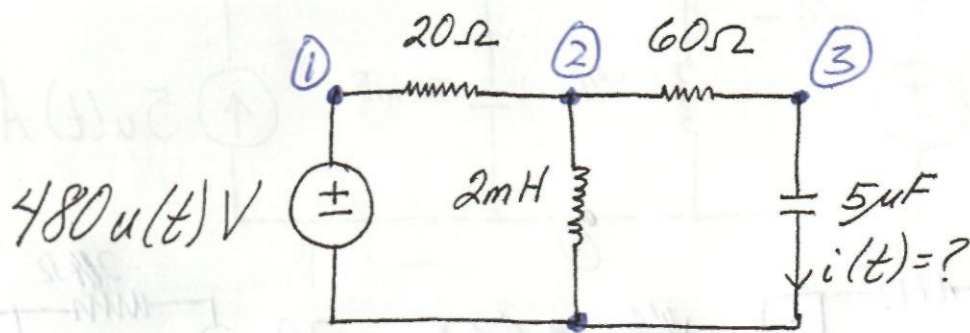
$$i(t) = j25 e^{-(0.6+j0.8)t} - j25 e^{-(0.6-j0.8)t}$$

$$i(t) = \underline{\underline{[50 e^{-0.6t} \sin(0.8t)] u(t) A}}$$

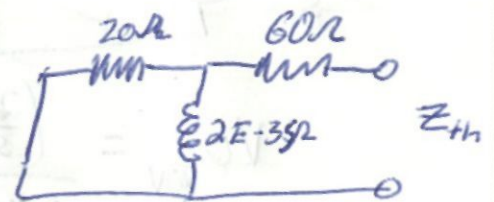
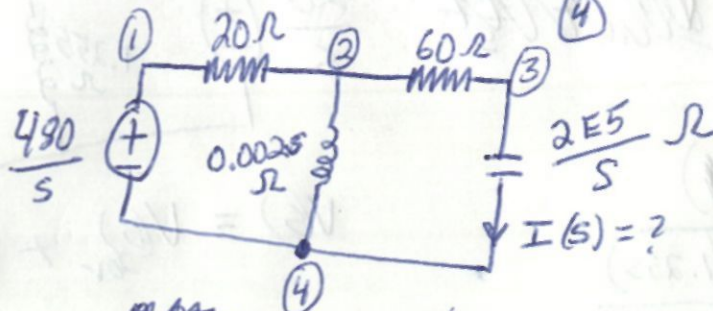
$u(t)$ is effective timestamp

second order should be 2nd degree polynomial = order (maybe greater)

Zero ICs
 $i(0) = 0A$
 $v(0) = 0V$
 $u(t) @ t < 0 = 0$



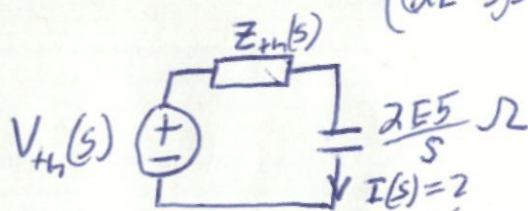
Use Thevenin equivalent



Voltage division

$$V_{th}(s) = \frac{\left(\frac{480}{s}\right)(2E-3/s)}{(2E-3/s) + 20}$$

$$Z_{th} = \frac{(2E-3/s)(20)}{20 + (2E-3/s)} + 60$$



$$I(s) = \frac{\left[\frac{480}{s}\right] \frac{(2E-3/s)}{(2E-3/s) + 20}}{\frac{2E-3/s \cdot 20}{20 + 2E-3/s} + [60] + \frac{2E5}{s}}$$

~~$$I(s) = \frac{480}{s}$$~~

~~$$I(s) = \frac{\left(\frac{480}{s}\right)(2E-3/s)(2E5)}{(2E-3/s + 20)(s)}$$~~

$$\frac{(2E-3/s)(20/s) + (60)(2E-3/s + 20)(s) + (2E5)(2E-3/s + 20)}{(2E-3/s + 20)(s)}$$

$$= \frac{(480)(2E-3)(2E5)}{0.04s^2 + 0.12s^2 + 1200s + 400s + 4E6} = \frac{192E3}{0.16s^2 + 1600s + 4E6}$$

$$= \frac{192E3}{0.16(s^2 + 1E4s + 25E6)}$$

some how

$$I(s) = \frac{6s}{s^2 + 10s + 25} = \frac{6s}{(s+5)^2}$$

$$\frac{6s}{(s+5)^2} = \frac{A}{s+5} + \frac{B}{(s+5)^2}$$

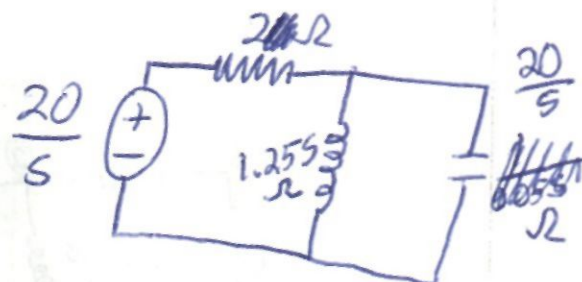
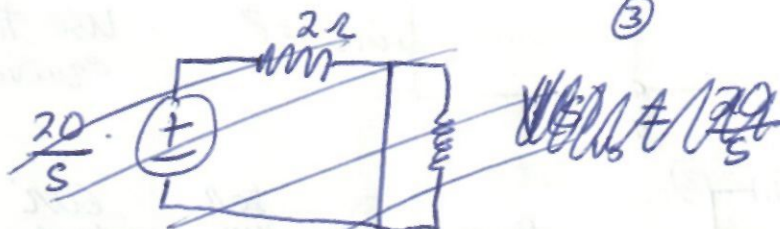
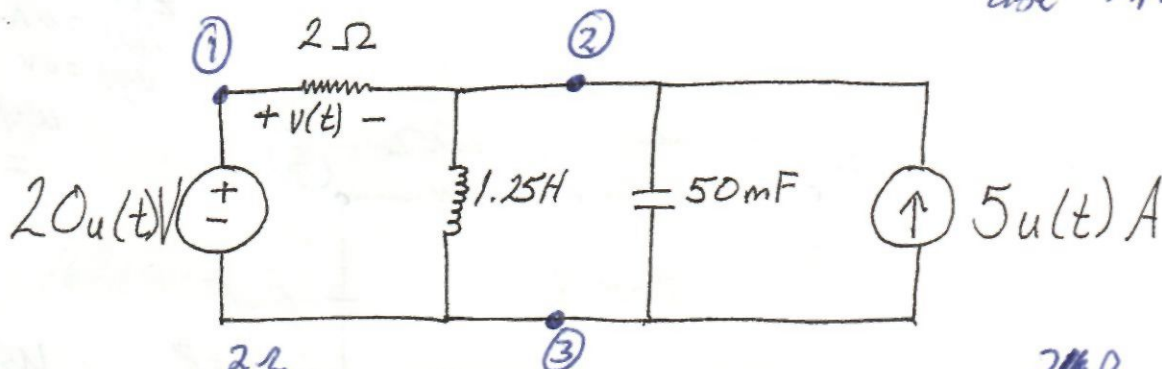
$$6s = A(s+5) + B$$

$$6(-5) = B = \underline{\underline{-30}}$$

Zero iCS

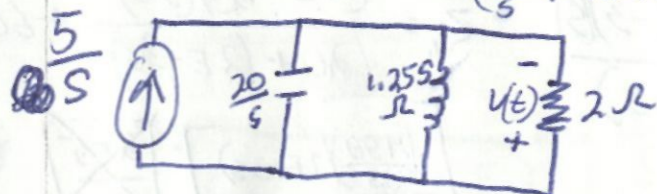
Use superposition

$$V = IZ$$



$$V(s)_{20V} = \frac{\left(\frac{20}{s}\right) \cdot (2)}{2 + \left(\frac{20}{s}\right)(1.25s)} = \frac{(20/s) \cdot (2)}{(20/s + 1.25s)}$$

$$V(s) = V(s)_{20V} + V(s)_{5A}$$



$$V(s)_{5A} = \frac{\left[\frac{5}{s}\right] \cdot \frac{[1.25s][20/s]}{[1.25s + 20/s]}}{2 + \frac{[1.25s][20/s]}{[1.25s + 20/s]}} \cdot (-2)$$

$$V(s) = \left(\frac{\frac{20}{s} \cdot 2}{2 + \frac{(20/s)(1.25s)}{(20/s + 1.25s)}} \right) + \left(\frac{\frac{5}{s} \cdot \frac{(1.25s)(20/s)}{[1.25s + 20/s]}}{2 + \frac{(20/s)(1.25s)}{(20/s + 1.25s)}} \right) = \frac{40/s + \frac{(6.25)(20)}{1.25s^2 + 20s}}{2 + \frac{25}{20/s + 1.25s}}$$

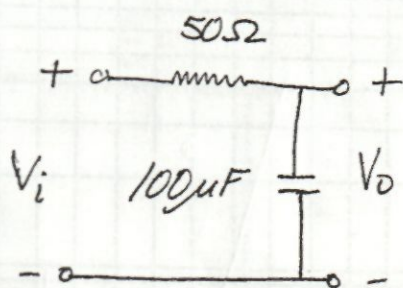
$$\dots = \frac{20s^2 - 100s + 320}{(s)(s+2)(s+8)} = 2 + \frac{25s}{20s + 1.25s^2}$$

check

$$\begin{aligned} A &= 20 \\ B &= -50 \\ C &= 50 \end{aligned}$$

$$V(t) = [20 - 50e^{-2t} + 50e^{-8t}] u(t)$$

Volts



What kind of filter is this?

$$H(s) = \frac{V_o(s)}{V_i(s)}$$

$$V_o(s) = \frac{(V_i(s)) \left(\frac{1}{sC} \right)}{\left(R + \frac{1}{sC} \right)}$$

$$H(s) = \frac{(V_i(s)) \left(\frac{1}{sC} \right)}{\left(R + \frac{1}{sC} \right)} \cdot \frac{1}{(V_i(s))} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} \cdot \frac{s}{s} = \frac{1}{sRC + 1}$$

$$H(s) = \frac{(1/C)}{(Rs + 1/C)} = \frac{1/R_C}{(s + 1/R_C)} \stackrel{\text{Sub in values (200)}}{=} \frac{1}{(s + 200)}$$

$$\frac{200}{\left(\frac{s}{200} + 1 \right) 200} = \frac{1}{\left(\frac{s}{200} + 1 \right)} \quad \text{let } s = j\omega \Rightarrow \frac{1}{\left(\frac{j\omega}{200} + 1 \right)} = H(j\omega)$$

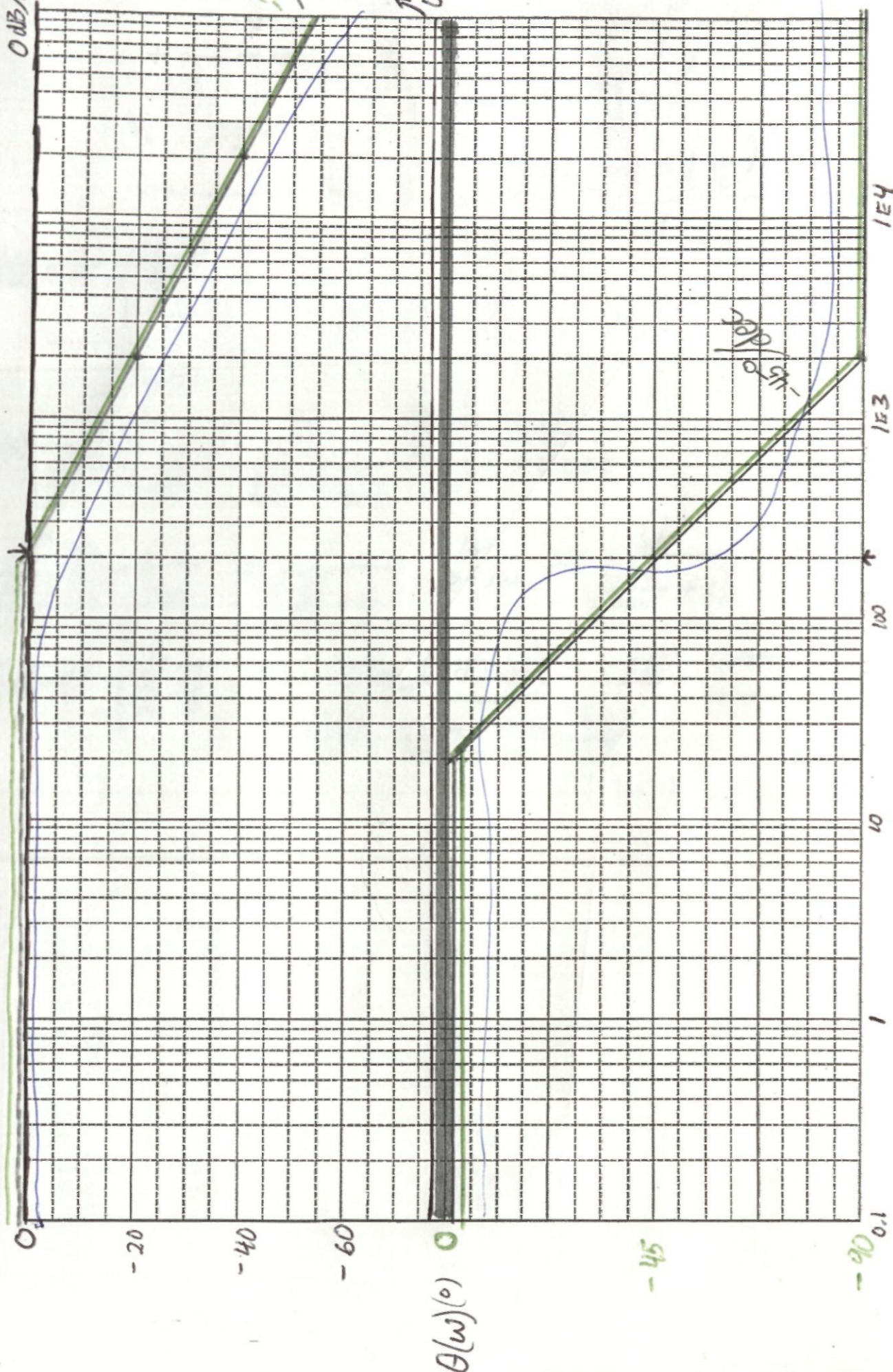
$$|H(j\omega)| = \frac{1}{\left| \frac{j\omega}{200} + 1 \right|} \Rightarrow \underline{20 \log_{10}(|H(j\omega)|)} = \underline{0 \text{ dB}} - \underline{20 \log_{10} \left(\left| \frac{j\omega}{200} + 1 \right| \right)}$$

$$\underline{\theta(\omega) = 0^\circ - \arctan \left(\frac{\omega}{200} \right)}$$

Low-Pass Filter

$|H(j\omega)|$ (dB)

0 dB/dec



$\theta(\omega)$ (°)

-45

-90

0°

10

100

10^3

10^4

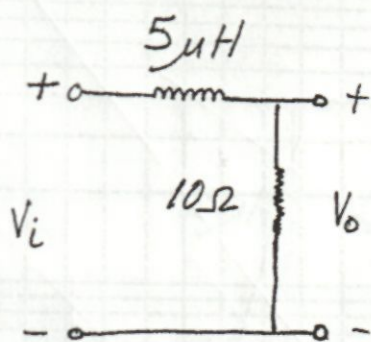
-45°

ω (rad/s)

0°

-20 dB/dec

What kind of filter is this?



Voltage Gain Transfer Function

$$H(s) = \frac{V_o(s)}{V_i(s)}$$

Voltage division

$$V_o(s) = \frac{(V_i(s))(R)}{(Ls + R)}$$

$$H(s) = \frac{(V_i(s))(R)}{(Ls + R)(V_i(s))}$$

$$H(s) = \frac{R}{(Ls + R)} = \frac{\frac{R}{L}}{(s + \frac{R}{L})}$$

$$H(s) = \frac{2E6}{(s + 2E6)}$$

$$= \frac{1}{(\frac{s}{2E6} + 1)}$$

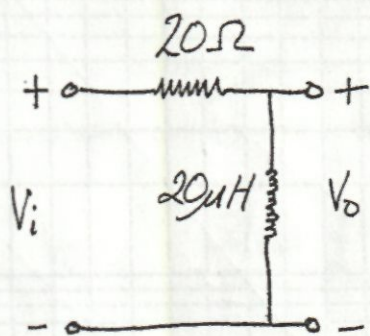
sub in values

$$H(j\omega) = \frac{1}{(\frac{j\omega}{2E6} + 1)}$$

$$|H(j\omega)| = \frac{|1|}{|\frac{j\omega}{2E6} + 1|}$$

$$20 \log_{10}(|H(j\omega)|) = 0dB - 20 \log_{10} \left(\left| \frac{j\omega}{2E6} + 1 \right| \right)$$

$$\theta(\omega) = 0^\circ - \arctan \left(\frac{\omega}{2E6} \right)$$



What kind of filter is this?

$$H(s) = \frac{V_o(s)}{V_i(s)}$$

$$V_o(s) = \frac{(V_i(s))(R)}{(R + \frac{1}{sC})}$$

$$H(s) = \frac{(V_i(s))(R)}{R + \frac{1}{sC}} \cdot \frac{1}{(V_i(s))}$$

$$H(s) = \frac{(R)}{(R + \frac{1}{sC})} \cdot \frac{s}{s}$$

$$= \frac{(Rs)}{(Rs + \frac{1}{C})} = \frac{R(s)}{R(s + \frac{1}{RC})}$$

$$H(s) = \frac{s}{s + \frac{1}{RC}} = \frac{(s)}{(s + 100)}$$

$$H(s) = \frac{s(0.01)}{(\frac{s}{100} + 1)}$$

$$H(j\omega) = \frac{j\omega \cdot 0.01}{(\frac{j\omega}{100} + 1)}$$

$$|H(j\omega)| = \frac{|0.01| |\omega|}{|\frac{j\omega}{100} + 1|}$$

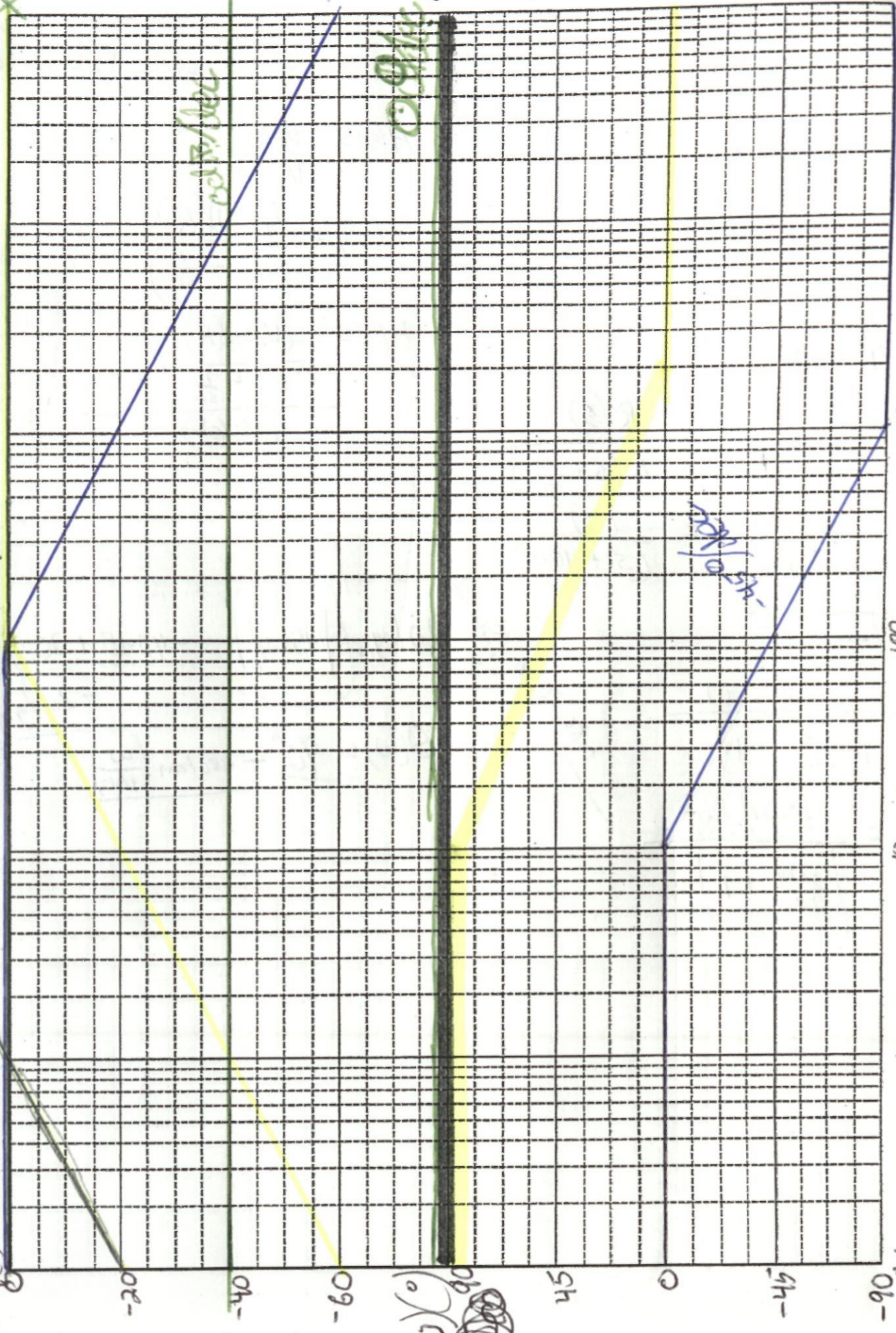
take 20 log10 of both sides

$$20 \log_{10}(|H(j\omega)|) = -40 \text{ dB} + 20 \log_{10}(|\omega|) - 20 \log_{10}(|\frac{j\omega}{100} + 1|)$$

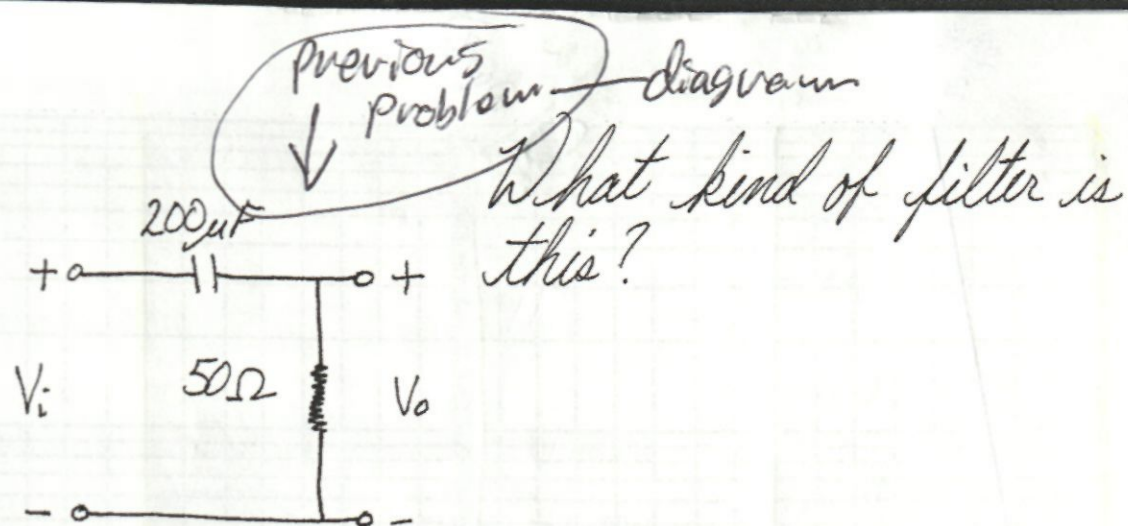
$$\theta(\omega) = 90^\circ - \arctan\left(\frac{\omega}{100}\right)$$

20dB/dec High-Pass Filter

$|H(\omega)|(\text{dB})$



$\phi(\omega)^\circ$



$$H(s) = \frac{V_o(s)}{V_i(s)} \quad \dots \quad V_o(s) = \frac{V_i(s)(Ls)}{(Ls + R)}$$

This work is for previous question diagram

$$H(s) = \frac{sL}{(sL + R)} = \frac{s}{s + \frac{R}{L}} = \frac{s}{(s + 1E6)}$$

$\frac{R}{L} = 1E6$

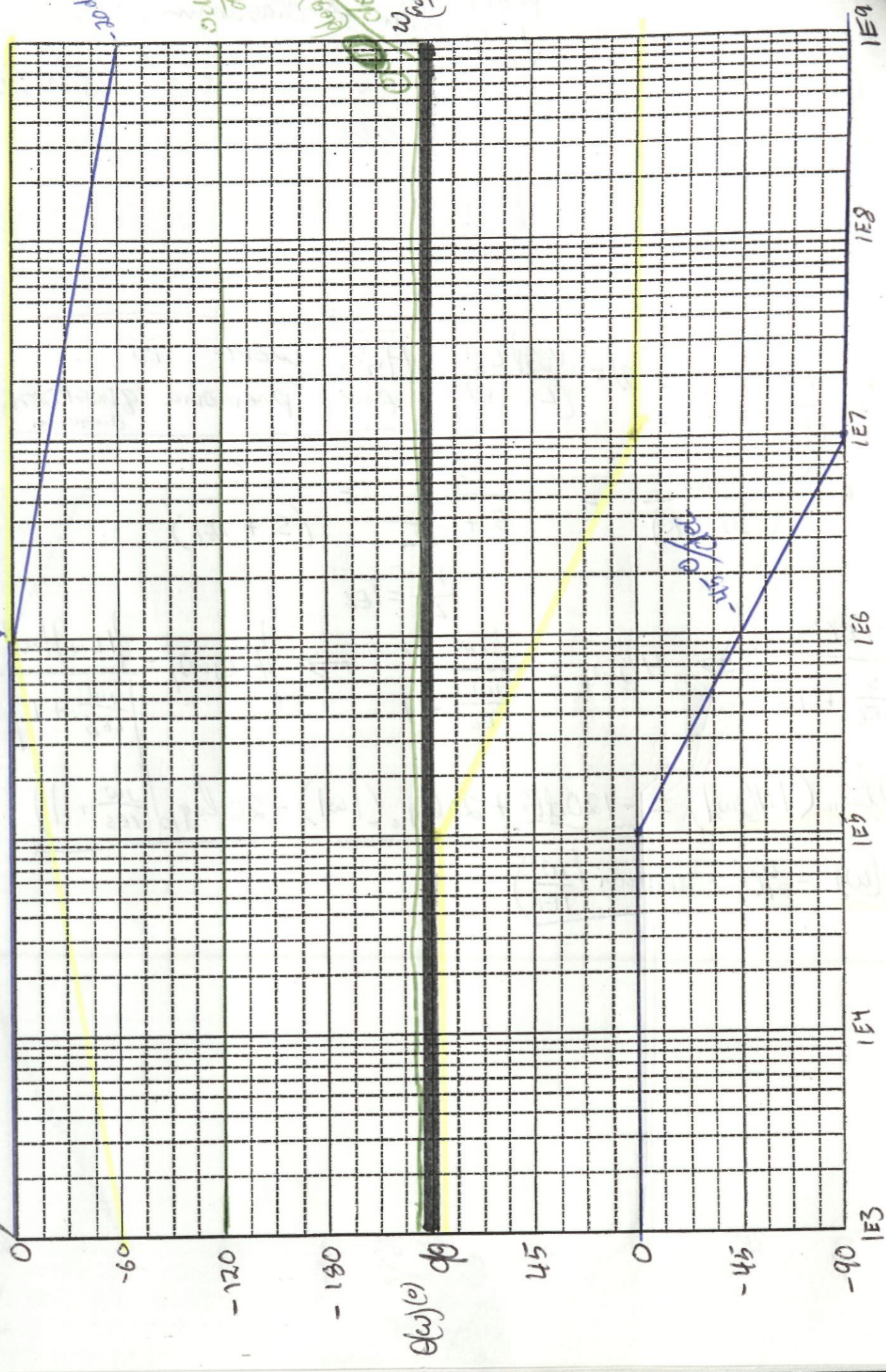
$$H(s) = \frac{(1E6)s}{\left(\frac{s}{1E6} + 1\right)} \Rightarrow H(j\omega) = \frac{j \cdot 1E6 \cdot \omega}{\frac{j\omega}{1E6} + 1} \rightarrow |H(j\omega)| = \frac{|1E6|\omega}{\left|\frac{j\omega}{1E6} + 1\right|}$$

$$20 \log_{10}(|H(j\omega)|) = -120dB + 20 \log_{10}(\omega) - 20 \log_{10}\left(\left|\frac{j\omega}{1E6} + 1\right|\right)$$

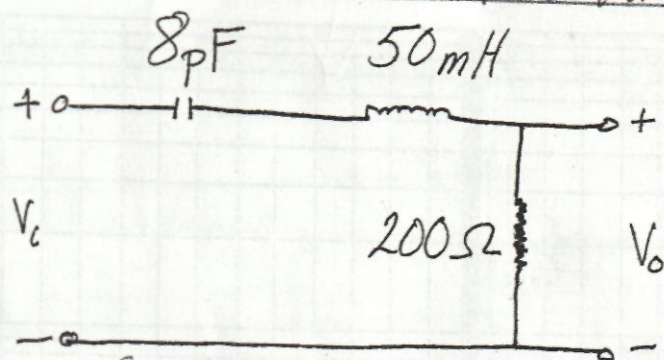
$$\theta(\omega) = 90^\circ - \arctan\left(\frac{\omega}{1E6}\right)$$

High-pass Filter

$|H(j\omega)|_{dB}$



Band Pass Filter



What kind of filter is this?

$$H(s) = \frac{V_o(s)}{V_i(s)}$$

$$V_o(s) = \frac{(V_i(s) \cdot R)}{Ls + R + \frac{1}{sC}} \Rightarrow H(s) = \frac{(V_i(s) \cdot R)}{Ls + R + \frac{1}{sC}}$$

$$H(s) = \frac{R}{Ls + R + \frac{1}{sC}} \cdot \frac{s}{s} \Rightarrow = \frac{SR}{Ls^2 + Rs + \frac{1}{C}}$$

$$H(s) = \frac{\left(\frac{R}{L}\right)(s)}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{\left(\frac{R}{L}\right)(s)}{(s + \omega_1)(s + \omega_2)}$$

$$H(s) = \frac{\frac{R}{L\omega_1\omega_2} \cdot s}{\left(\frac{s}{\omega_1} + 1\right)\left(\frac{s}{\omega_2} + 1\right)} \Rightarrow H(j\omega) = \frac{j\left(\frac{R}{L\omega_1\omega_2}\right)\omega}{\left(\frac{j\omega}{\omega_1} + 1\right)\left(\frac{j\omega}{\omega_2} + 1\right)}$$

$$|H(j\omega)| = \frac{\left|\frac{R}{L\omega_1\omega_2}\right| |\omega|}{\left|\frac{j\omega}{\omega_1} + 1\right| \left|\frac{j\omega}{\omega_2} + 1\right|}$$

$$20 \log_{10}(|H(j\omega)|) = 20 \log_{10}\left(\left|\frac{R}{L\omega_1\omega_2}\right|\right) + 20 \log_{10}(|\omega|) - 20 \log_{10}\left|\frac{j\omega}{\omega_1} + 1\right| - 20 \log_{10}\left|\frac{j\omega}{\omega_2} + 1\right|$$

$$\phi(\omega) = 90^\circ - \arctan\left(\frac{\omega}{\omega_1}\right) - \arctan\left(\frac{\omega}{\omega_2}\right)$$

$$K_0 = 20 \log_{10}\left(\left|\frac{R}{L\omega_1\omega_2}\right|\right)$$

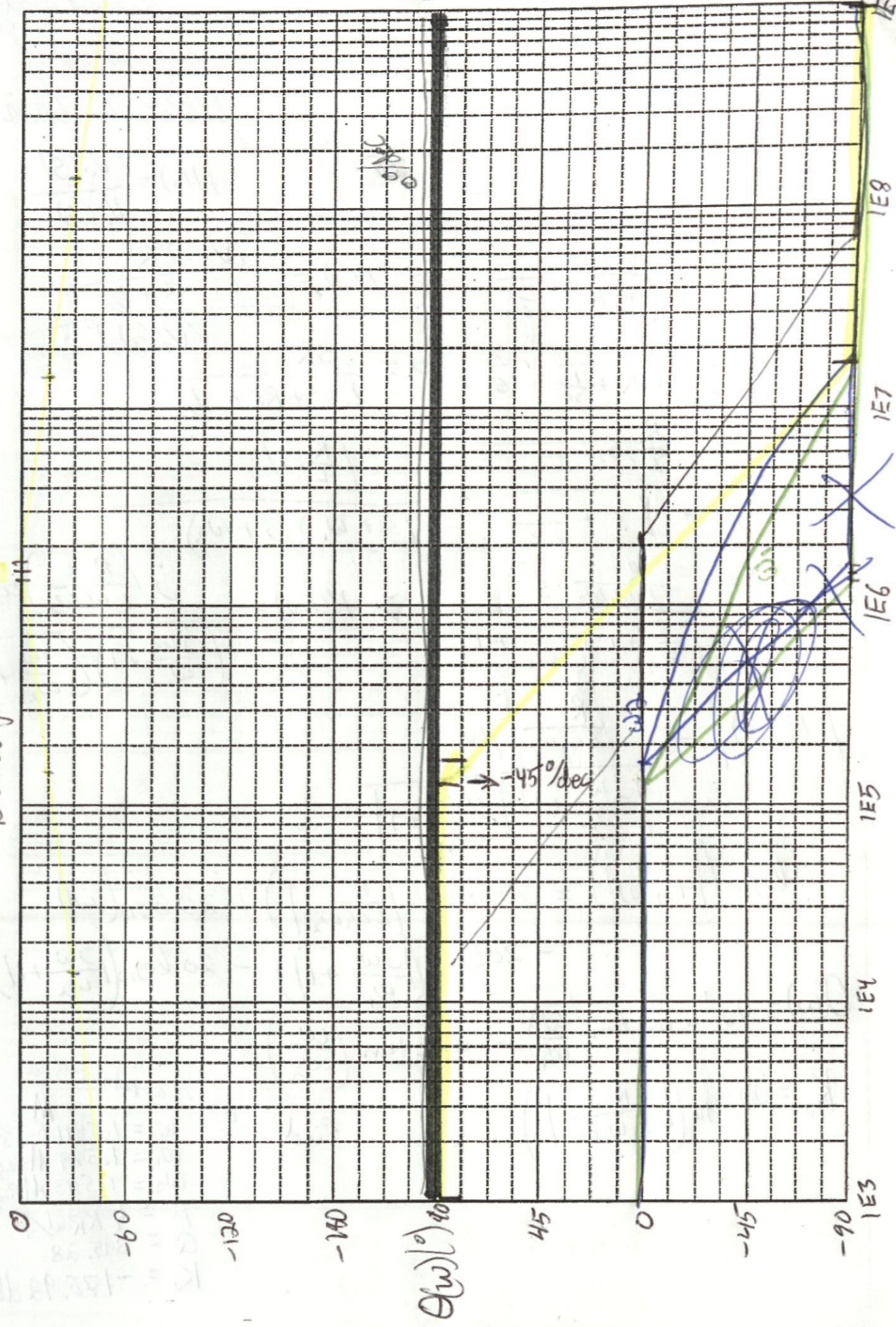
Find

characteristics

- $\omega_0 = 1.5811 \text{ Mrad/s}$
- $\omega_1 = 1.579 \text{ Mrad/s}$
- $\omega_2 = 1.583 \text{ Mrad/s}$
- $B = 4 \text{ Krad/s}$
- $Q = 395.28$
- $K_0 = -175.92 \text{ dB}$

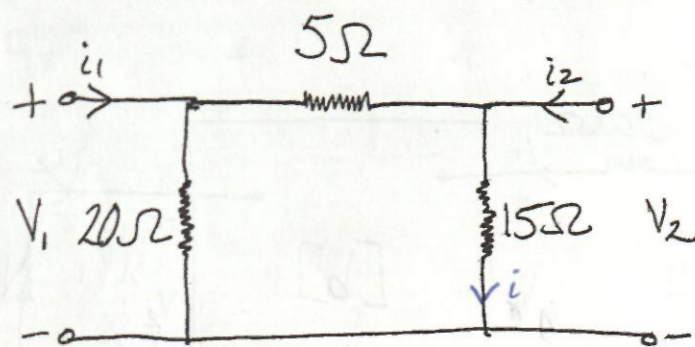
Band pass filter

$|H(j\omega)| (dB)$



ω rad/sec

Calculate the z -parameters of this two-port network.



$$Z_{11} = \frac{V_1}{i_1} \Big|_{i_2=0} \Rightarrow Z_{11} = \frac{10i_1}{i_1} \Rightarrow Z_{11} = 10\Omega$$

$$Z_{12} = \frac{V_1}{i_2} \Big|_{i_1=0} \Rightarrow Z_{12} = \frac{20\Omega \cdot \frac{15}{40} i_2}{i_2} \Rightarrow Z_{12} = 7.5\Omega$$

$$Z_{21} = \frac{V_2}{i_1} \Big|_{i_2=0} \Rightarrow Z_{21} = \frac{(15\Omega) \left(\frac{20i_1}{40} \right)}{i_1} \Rightarrow Z_{21} = 7.5\Omega$$

$$Z_{22} = \frac{V_2}{i_2} \Big|_{i_1=0} \Rightarrow Z_{22} = \frac{(15)(25)\Omega}{(20+15)} = Z_{22} = 9.375\Omega$$

Calculate the y -parameters of the same network

$$y_{11} = 0.25 \text{ V (mhos)}$$

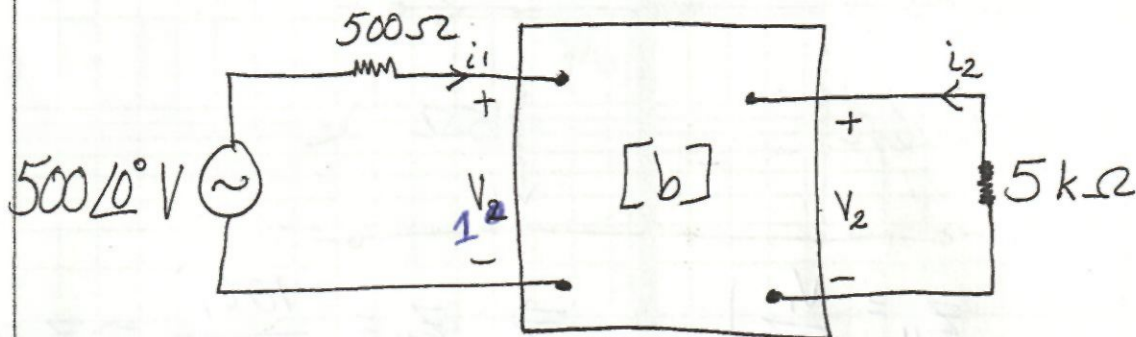
$$y_{12} = -0.2 \text{ V}$$

$$y_{21} = -0.2 \text{ V}$$

$$y_{22} = \frac{Z_{11}}{Z_{11}Z_{22} - Z_{12}Z_{21}} = 0.26 \text{ V}$$

Convert Z values to y parameters using table 18.1

Determine $\bar{V}_2$, $P_{L,avg}$, $P_{in,avg}$, $\bar{Z}_L$ (max. power transfer), & $P_{L,avg,max}$.



$$b_{11} = -20 \quad b_{12} = -3k\Omega$$

$$b_{21} = -2mV \quad b_{22} = -0.2$$

$$\frac{V_2}{V_g} =$$

~~$$\frac{V_2}{V_g} = 356.88$$~~

~~$$V_2 = 356.88 V$$~~

$$V_2 = 263.16 \angle 0^\circ V$$

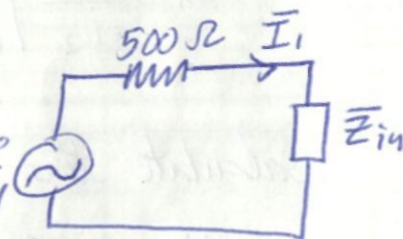
$$\frac{V_2}{V_g} = \frac{(b_{11}b_{22} - b_{12}b_{21})\bar{Z}_L}{b_{12} + b_{11}\bar{Z}_g + b_{22}\bar{Z}_L + b_{21}\bar{Z}_g\bar{Z}_L}$$

$$P_{L,avg} = \left(\frac{1}{2}\right) \frac{\bar{V}_2^2}{|\bar{Z}_L|} = 6.92 W$$

$$P_{in,avg} = \left(\frac{1}{2}\right) (|\bar{I}|^2) (\bar{Z}_{in}) = 41.55 W$$

$$\bar{Z}_L = \bar{Z}_{Th}^* \Rightarrow \bar{Z}_L = 10.83 k\Omega$$

$$P_{L,avg,max} = 8.01 W$$



$$Z_{in} = 133.33 \Omega$$

↑
on table