TWO-PORT NETWORK

TKT 419/3

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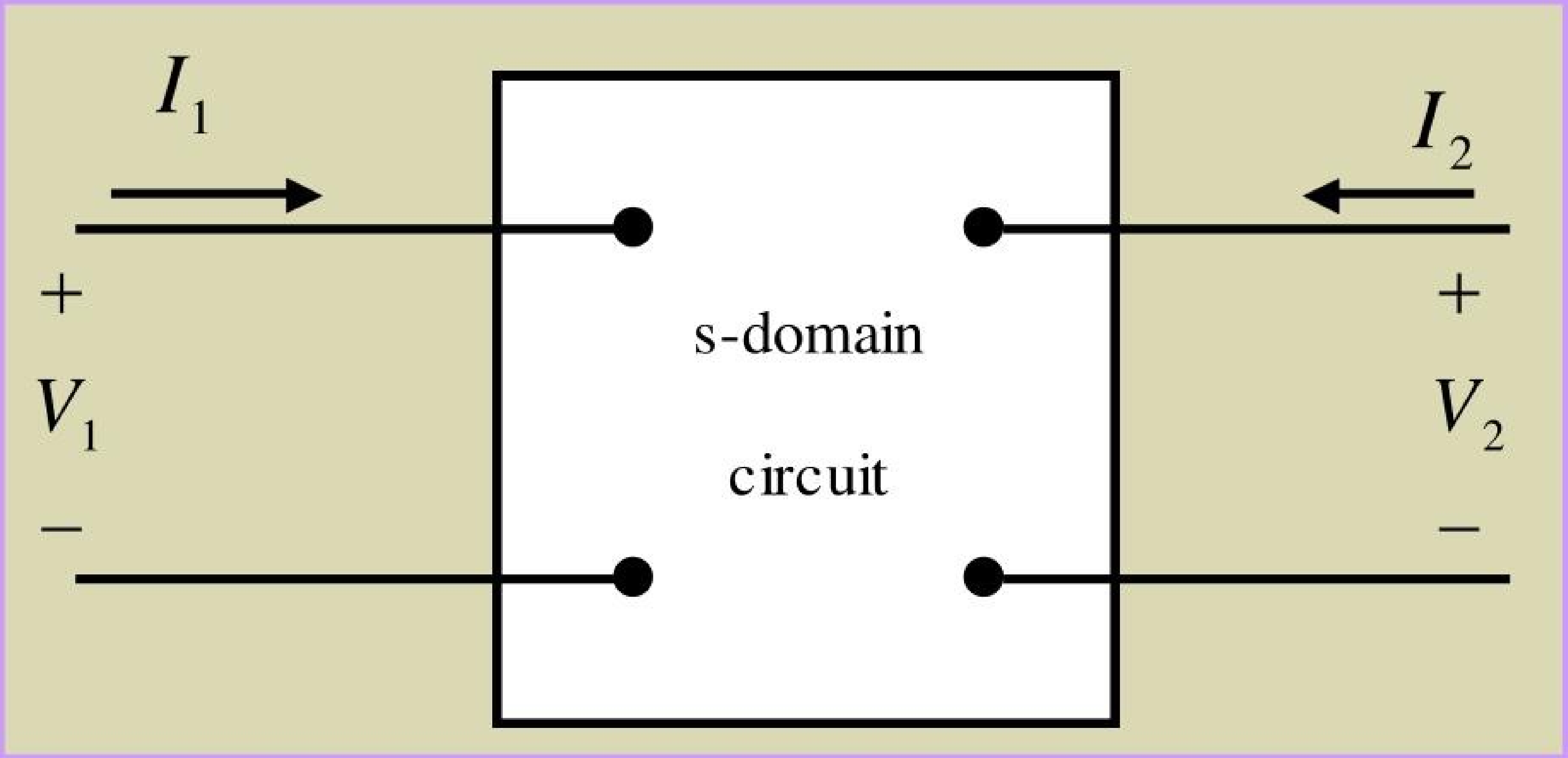
TWO-PORT NETWORK

- ***Terminal equations***
- Two-port parameters
- Relationships among two-port parameters
- Analysis of terminated two-port circuit
- Interconnected two-port circuits

TERMINAL EQUATIONS



s-domain two-port basic building block

Terminal equations relationships

- 1st Relationship:

$$V_1 = z_{11} I_1 + z_{12} I_2$$

$$V_2 = z_{21} I_1 + z_{22} I_2$$

2nd Relationship:

$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

3rd Relationship:

$$V_1 = a_{11}V_2 - a_{12}I_2$$

$$I_1 = a_{21}V_2 - a_{22}I_2$$

4^{th} Relationship:

$$V_2 = b_{11}V_1 - b_{12}I_1$$

$$I_2 = b_{21}V_1 - b_{22}I_1$$

5th Relationship:

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

6th Relationship:

$$I_1 = g_{11}V_1 + g_{12}I_2$$

$$V_2 = g_{21}V_1 + g_{22}I_2$$

TWO PORT NETWORK:

- Terminal equations
- ***Two-port parameters***
- Relationships between parameters
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IMPEDANCE PARAMETERS

- **z parameters**

$$V_1 = z_{11} I_1 + z_{12} I_2$$
$$V_2 = z_{21} I_1 + z_{22} I_2$$

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad \Omega \qquad z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad \Omega$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \quad \Omega \qquad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \quad \Omega$$

ADMITTANCE PARAMETERS

y parameters

$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad S \qquad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \quad S$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \quad S \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} \quad S$$

a parameters

$$a_{11} = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad a_{12} = -\left.\frac{V_1}{I_2}\right|_{V_2=0} \quad \Omega$$

$$a_{21} = \left.\frac{I_1}{V_2}\right|_{I_2=0} \quad S \qquad a_{22} = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$$

b parameters

$$b_{11} = \frac{V_2}{V_1}\bigg|_{I_1=0} \qquad b_{12} = -\frac{V_2}{I_1}\bigg|_{V_1=0} \quad \Omega$$

$$b_{21} = \frac{I_2}{V_1}\bigg|_{I_1=0} \quad S \qquad b_{22} = -\frac{I_2}{I_1}\bigg|_{V_2=0}$$

a and b parameters are called transmission parameters

HYBRID PARAMETERS

h parameters

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \quad \Omega \qquad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} \quad S$$

INVERSE HYBRID PARAMETERS

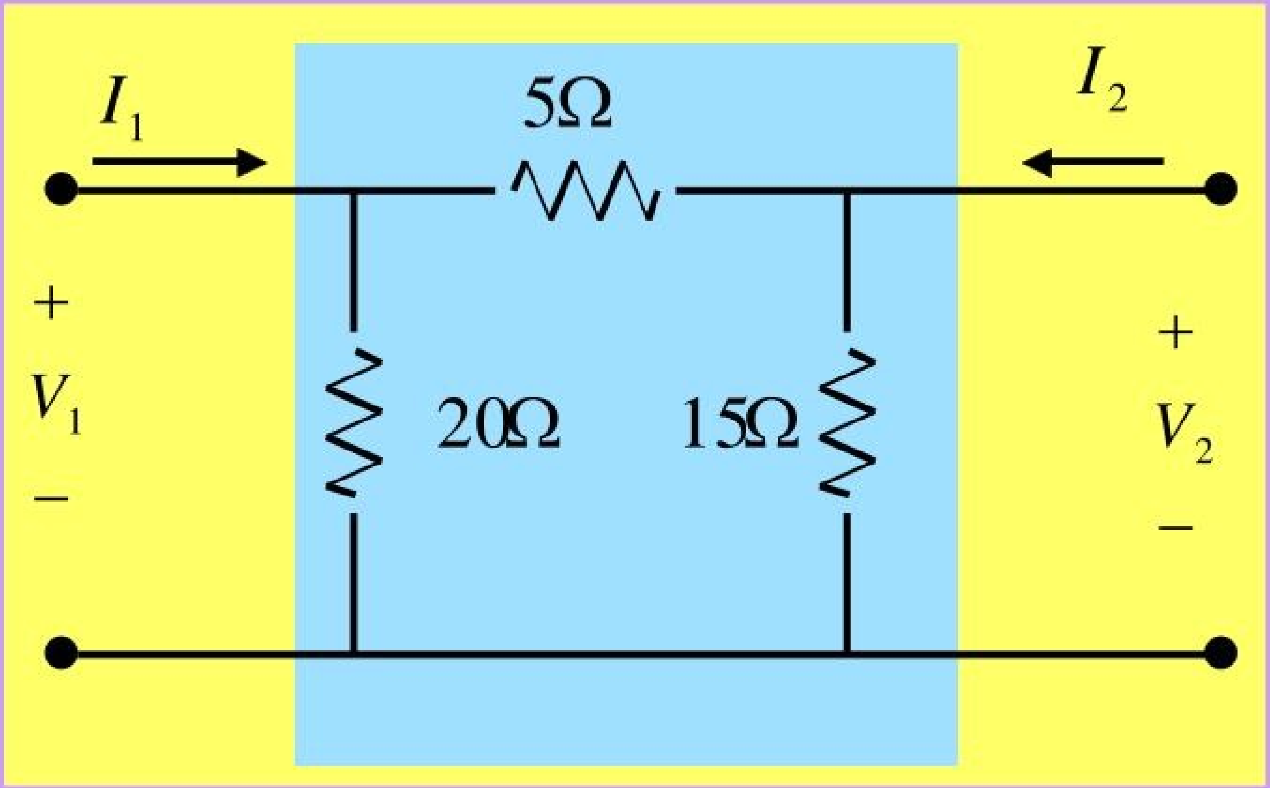
g parameters

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} \quad S \qquad g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}$$

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0} \qquad g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0} \quad \Omega$$

Example 1

- Find z parameter for the below circuit:



Solutions:

- When port 2 open-circuit, $I_2=0$

so,

$$z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} = \frac{20(20)}{40} = 10\Omega$$

- When $I_2=0$, thus V_2

$$V_2 = \frac{15}{15+5}V_1 = 0.75V_1$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{0.75V_1}{V_1/10} = 7.5\Omega$$

Now, when I_1=0,so

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{15(25)}{40} = 9.375\Omega$$

When port 1 open-circuit, $I_1=0$, so:

$$V_1 = \frac{20}{5+20}V_2 = 0.8V_2$$

$$I_2 = \frac{V_2}{9.375}$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{0.8V_2}{V_2/9.375} = 7.5\Omega$$

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RELATIONSHIPS BETWEEN PARAMETERS

$$V_1 = \frac{\begin{vmatrix} I_1 & y_{12} \\ I_2 & y_{22} \end{vmatrix}}{\begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{vmatrix}} = \frac{y_{22} I_1}{\Delta y} - \frac{y_{12} I_2}{\Delta y}$$

$$V_2 = \frac{\begin{vmatrix} y_{11} & I_1 \\ y_{21} & I_2 \end{vmatrix}}{\begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{vmatrix}} = \frac{y_{11} I_2}{\Delta y} - \frac{y_{21} I_1}{\Delta y}$$

Compare these two above equations:

$$z_{11} = \frac{y_{22}}{\Delta y} \qquad z_{12} = -\frac{y_{12}}{\Delta y}$$

$$z_{21} = -\frac{y_{21}}{\Delta y} \qquad z_{22} = \frac{y_{11}}{\Delta y}$$

Find z parameters as functions of a parameters

$$V_2 = \frac{1}{a_{21}} I_1 + \frac{a_{22}}{a_{21}} I_2$$

$$V_1 = \frac{a_{11}}{a_{21}} I_1 + \left(\frac{a_{11}a_{22}}{a_{21}} - a_{12} \right) I_2$$

Therefore,

$$z_{11} = \frac{a_{11}}{a_{21}} \qquad z_{12} = \frac{\Delta a}{a_{21}}$$

$$z_{21} = \frac{1}{a_{21}} \qquad z_{22} = \frac{a_{22}}{a_{21}}$$

RECIPROCAL TWO-PORT CIRCUITS

- If a two-port circuit is reciprocal, the following relationships exist among port parameters:

$$z_{12} = z_{21} \qquad y_{12} = y_{21}$$

$$a_{11}a_{22} - a_{12}a_{21} = \Delta a = 1$$

$$b_{11}b_{22} - b_{12}b_{21} = \Delta b = 1$$

$$h_{12} = -h_{21} \qquad g_{12} = -g_{21}$$

Symmetric of a reciprocal two-port circuits

- Following relationships exist among port parameters:

$$z_{11} = z_{22} \qquad y_{11} = y_{22}$$

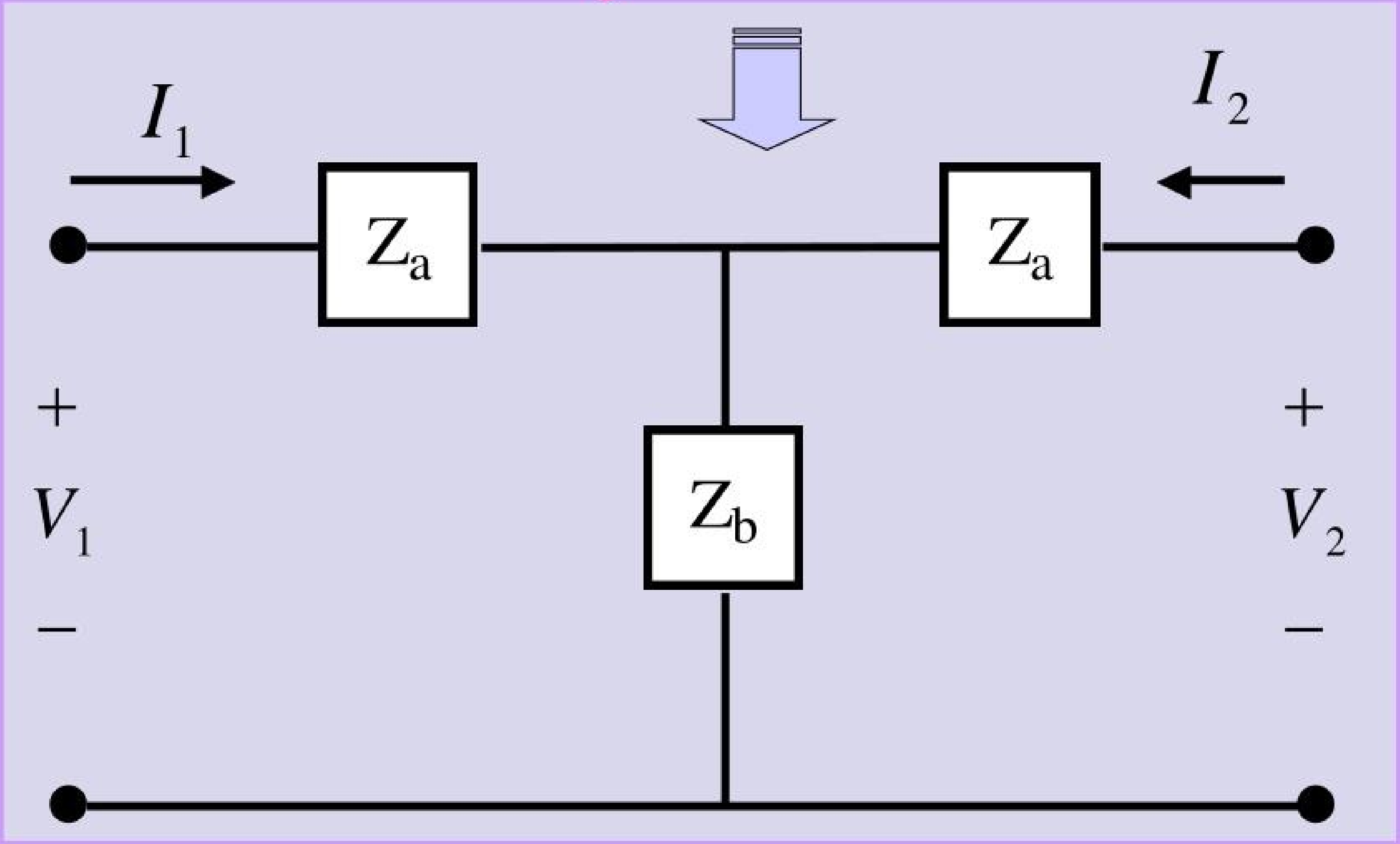
$$a_{11} = a_{22} \qquad b_{11} = b_{22}$$

$$h_{11}h_{22} - h_{12}h_{21} = \Delta h = 1$$

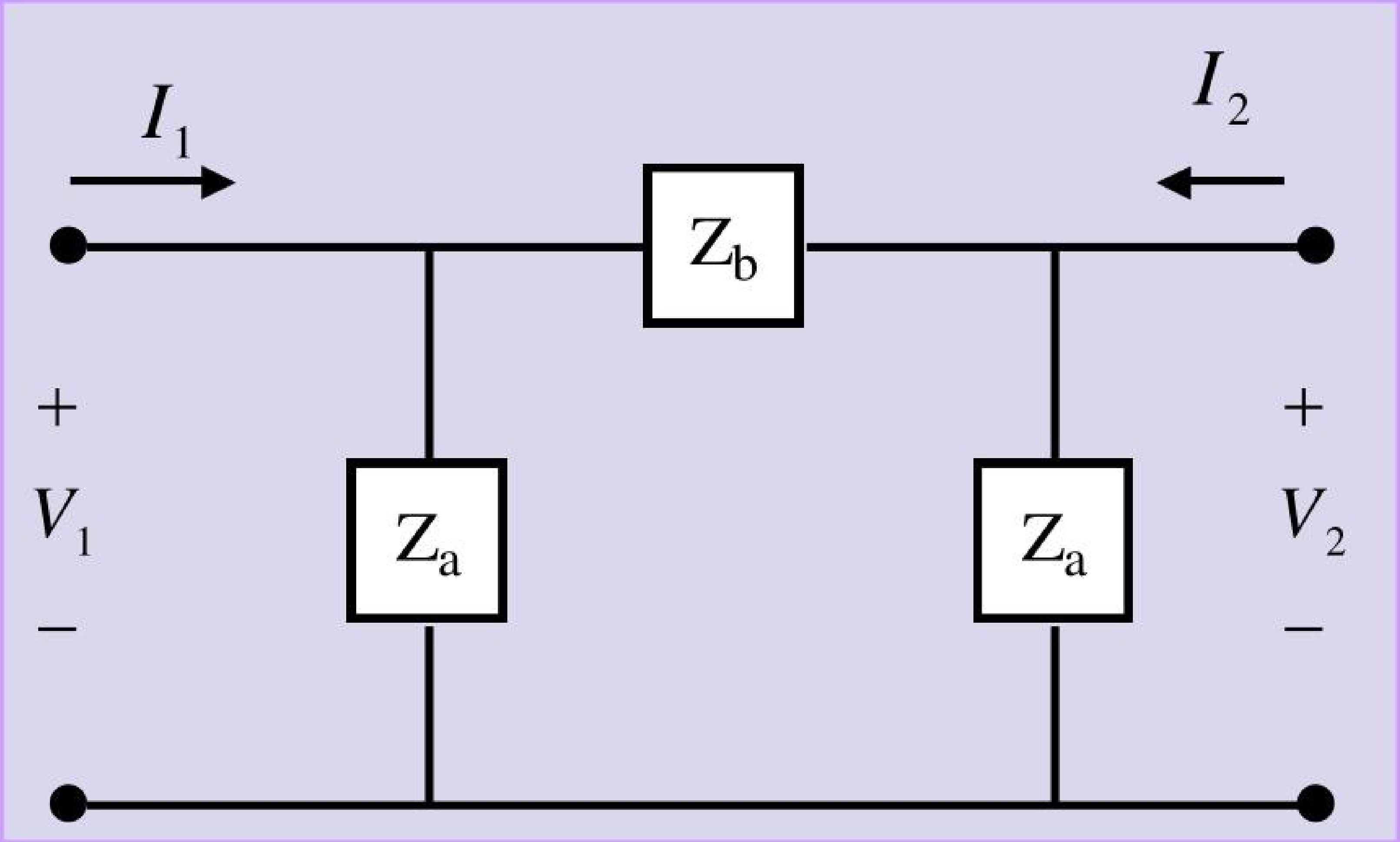
$$g_{11}g_{22} - g_{12}g_{21} = \Delta g = 1$$

Example of symmetric two-port circuits

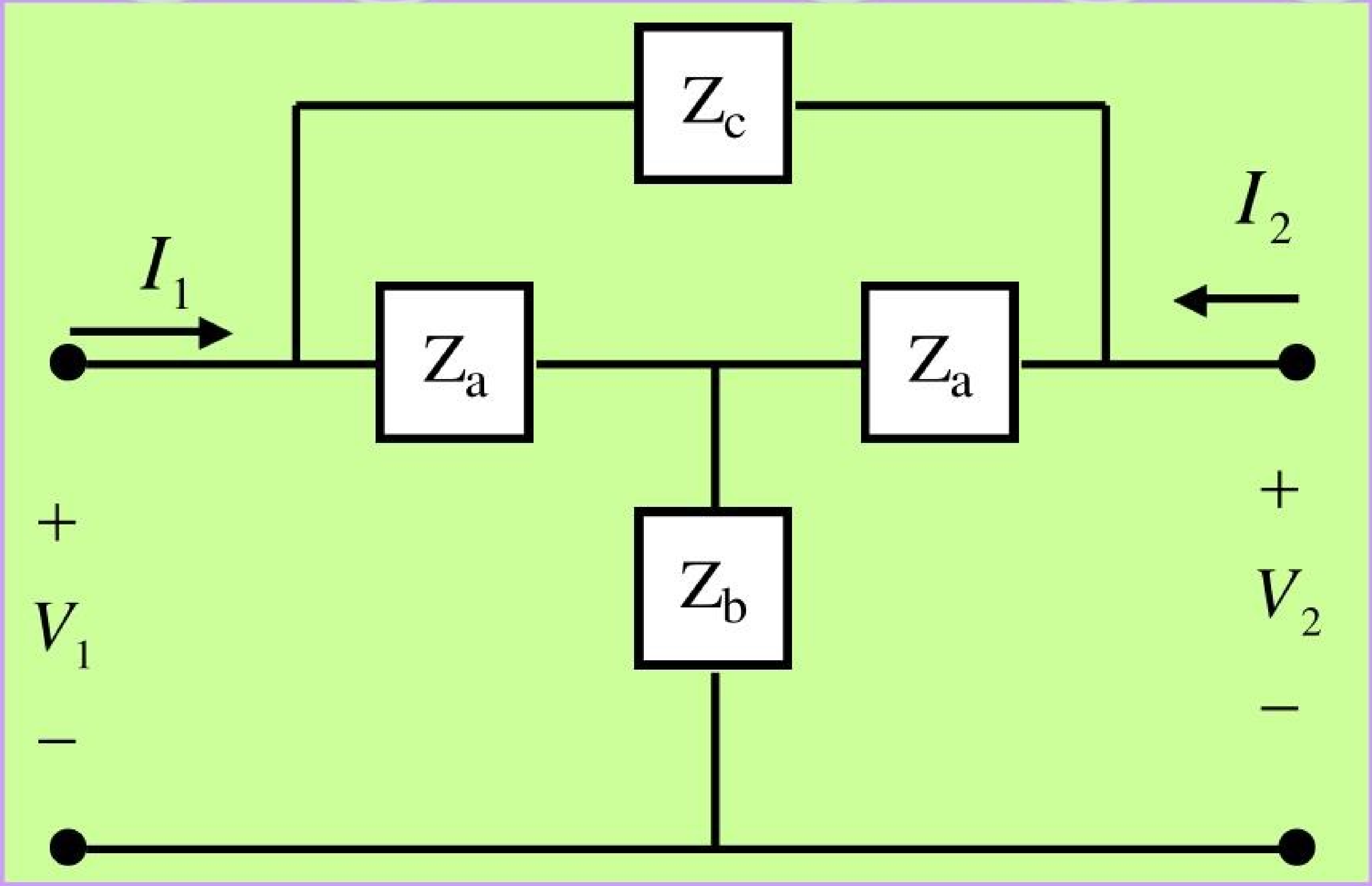
Symmetric tee



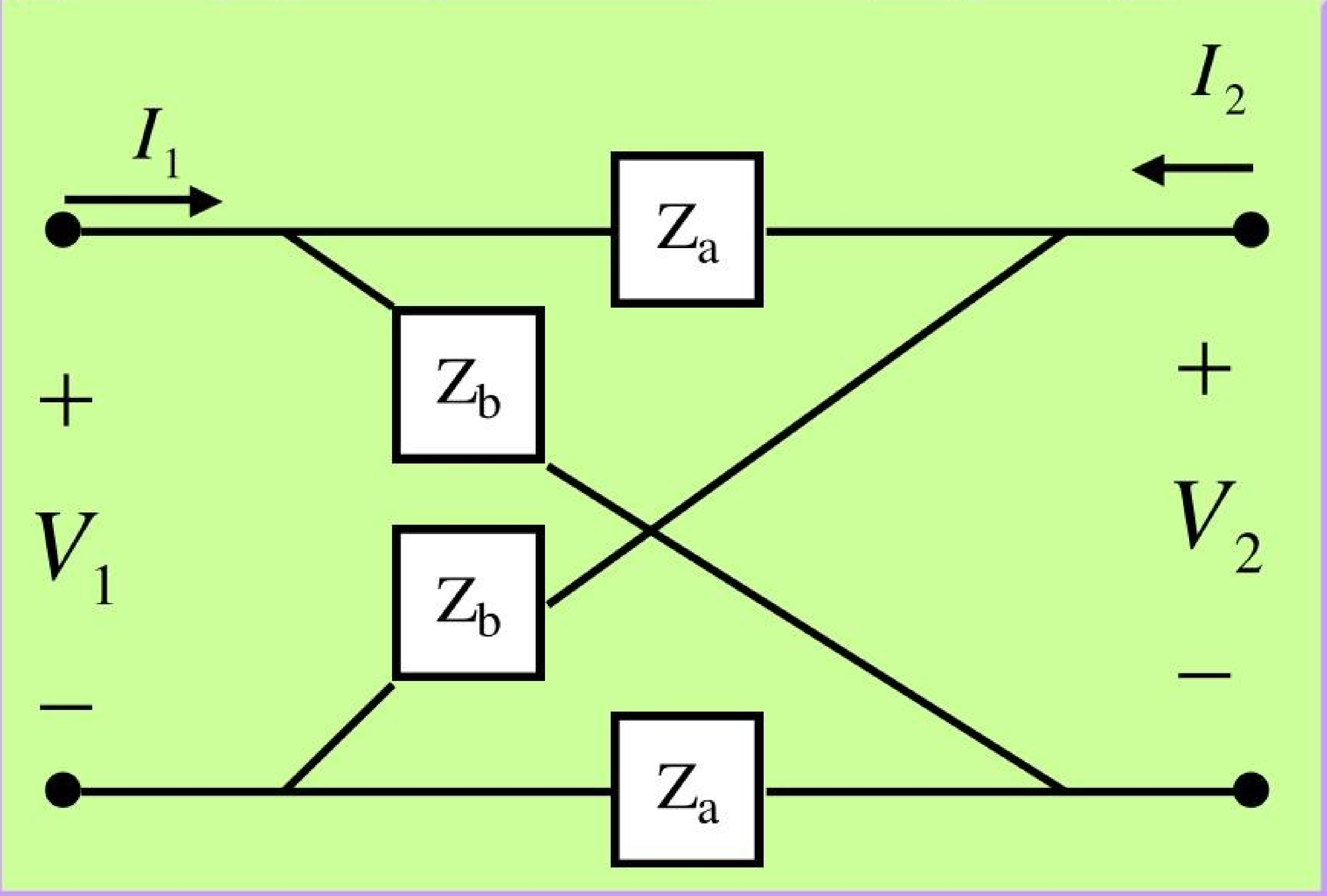
Symmetric pi



Symmetric bridge tee



Symmetric lattice



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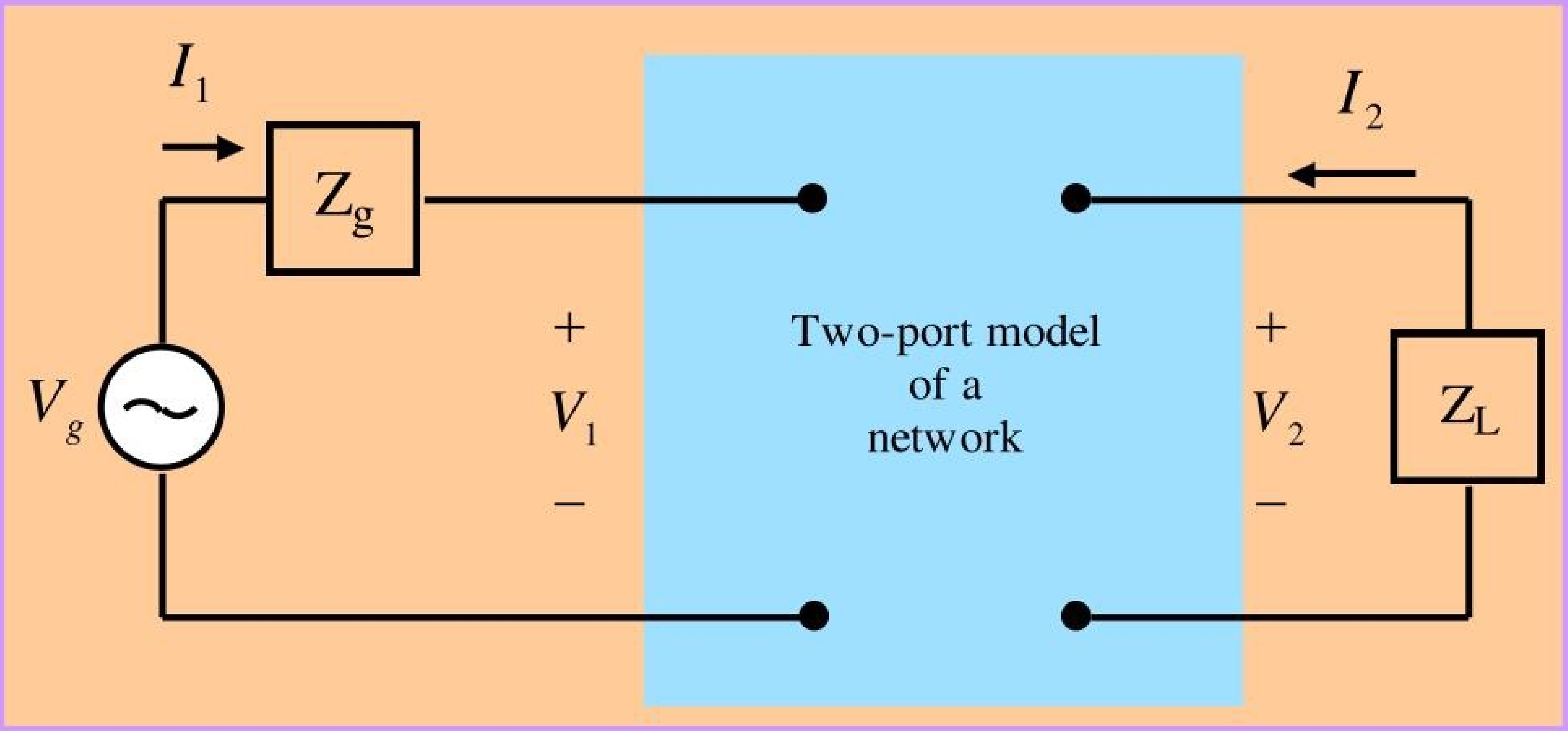
ANALYSIS OF TERMINATED TWO-PORT CIRCUITS



6 characteristics of terminated two-port circuit

- Input impedance ($Z_{in}=V_1/I_1$) or admittance ($Y_{in}=I_1/V_1$)
- Output current, I_2
- Thevenin voltage and impedance (Z_{Th}, V_{Th}) with respect to port 2
- Current gain I_2/I_1
- Voltage gain V_2/V_1
- Voltage gain V_2/V_g

6 characteristics in term of *z parameters*

- 4 parameter equations that describe the circuit:

$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

$$V_1 = V_g - I_1Z_g$$

$$V_2 = -I_2Z_L$$

1st characteristic (input impedance)

$$I_2 = \frac{-z_{21} I_1}{Z_L + z_{22}}$$

$$Z_{in} = z_{11} - \frac{z_{12} z_{21}}{z_{22} + Z_L}$$

2^{nd} characteristic (output current, I_2)

$$I_1 = \frac{V_g - z_{12} I_2}{z_{11} + Z_g}$$

$$I_2 = \frac{-z_{21} V_g}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12} z_{21}}$$

3rd characteristic (Thevenin voltage @ impedance)

$$V_2\big|_{I_2=0} = z_{21} I_1 = z_{21}\frac{V_1}{z_{11}}$$

$$V_2\big|_{I_2=0} = V_{Th} = V_g\,\frac{z_{21}}{Z_g + z_{11}}$$

Impedance Thevenin

$$I_1 = \frac{-z_{12} I_2}{z_{11} + Z_g}$$

$$\left.\frac{V_2}{I_2}\right|_{V_g = 0} = Z_{Th} = z_{22} - \frac{z_{12} z_{21}}{z_{11} + Z_g}$$

4th characteristic (current gain)

$$\frac{I_2}{I_1} = \frac{-z_{21}}{Z_L + z_{22}}$$

5^{th} characteristic (voltage gain V_2/V_1)

$$V_2 = z_{21} I_1 + z_{22}\left(\frac{-V_2}{Z_L}\right)$$

$$z_{11} I_1 = V_1 - z_{12}\left(\frac{-V_2}{Z_L}\right)$$

$$I_1 = \frac{V_1}{z_{11}} + \frac{z_{12} V_2}{z_{11} Z_L}$$

$$\frac{V_2}{V_1} = \frac{z_{21}Z_L}{z_{11}Z_L + z_{11}z_{22} - z_{12}z_{21}}$$

$$= \frac{z_{21}Z_L}{z_{11}Z_L + \Delta z}$$

6th characteristic (voltage gain V_2/V_g)

$$I_1 = \frac{z_{12}V_2}{Z_L(z_{11}+Z_g)} + \frac{V_g}{z_{11}+Z_g}$$

$$V_2 = \frac{z_{21}z_{12}V_2}{Z_L(z_{11}+Z_g)} + \frac{z_{21}V_g}{z_{11}+Z_g} - \frac{z_{22}}{Z_L}V_2$$

$$\frac{V_2}{V_g} = \frac{z_{21} Z_L}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12} z_{21}}$$

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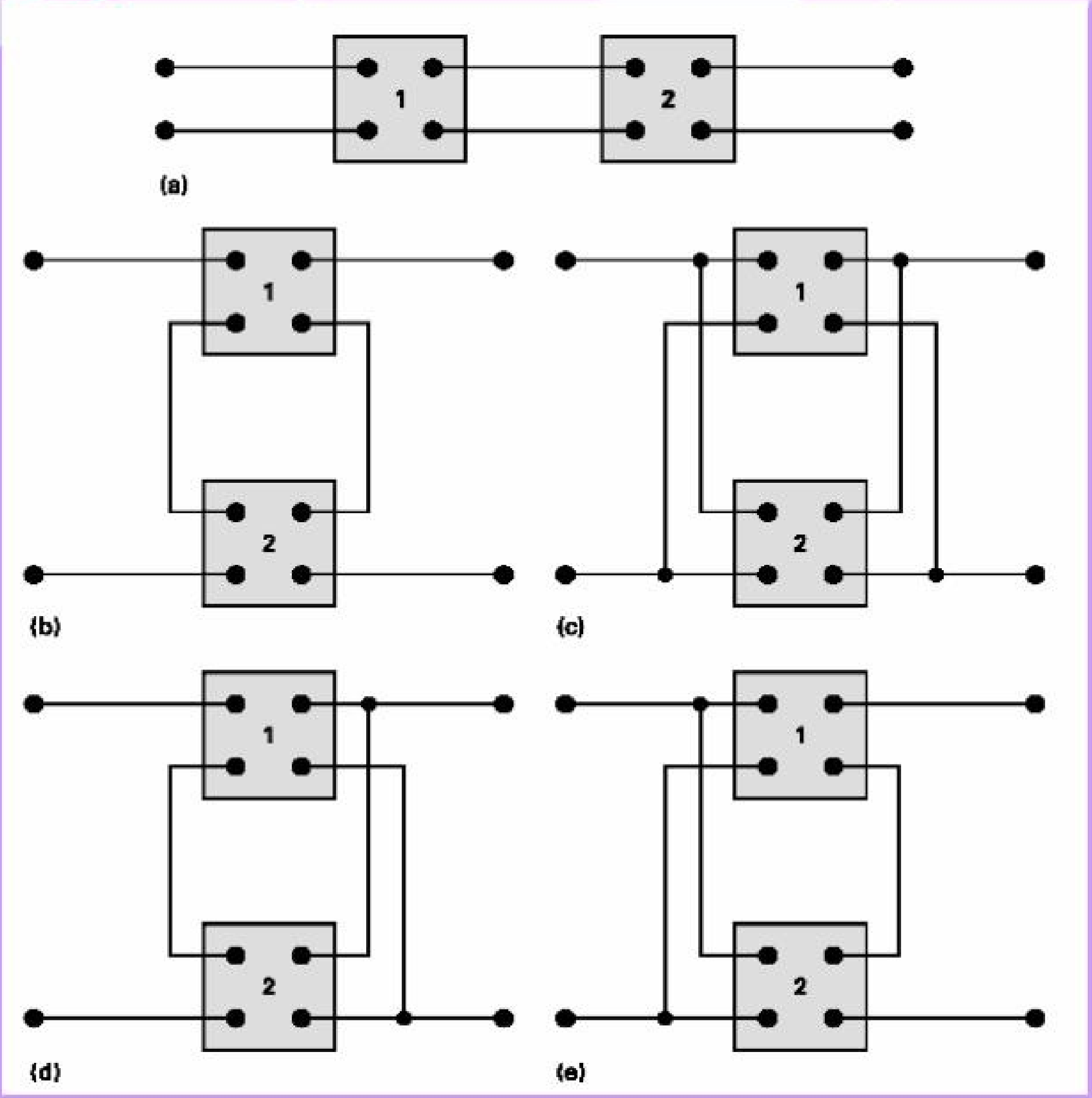
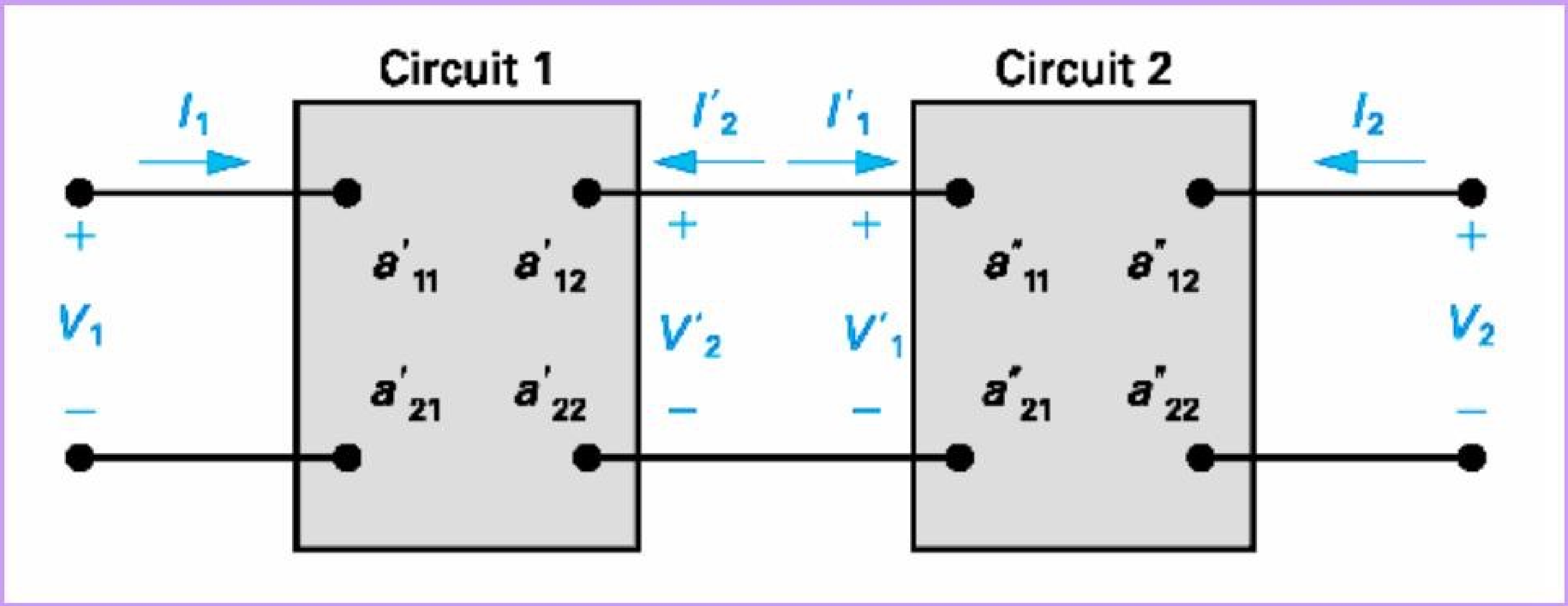
INTERCONNECTED TWO-PORT CIRCUITS



Cascade connection



Two-port parameters

$$V_1 = a_{11}V_2 - a_{12}I_2$$

$$I_1 = a_{21}V_2 - a_{22}I_2$$

$$V_1 = a'_{11}V'_2 - a'_{12}I'_2$$

$$I_1 = a'_{21}V'_2 - a'_{22}I'_2$$

Interconnection ($V'_2 = V'_1$ and $I'_2 = -I'_1$)

$$V_1 = a'_{11} V'_1 + a'_{12} I'_1$$

$$I_1 = a'_{21} V'_1 + a'_{22} I'_1$$

Relationship V'_1 and I'_1 with V2 and I2 in second circuit :

$$V_1' = a''_{11}V_2 - a''_{12}I_2$$

$$I_1' = a''_{21}V_2 - a''_{22}I_2$$

$$V_1 = (a'_{11}a''_{11} + a'_{12}a''_{21})V_2 - (a'_{11}a''_{12} + a'_{12}a''_{22})I_2$$

$$I_1 = (a'_{21}a''_{11} + a'_{22}a''_{21})V_2 - (a'_{21}a''_{12} + a'_{22}a''_{22})I_2$$

$$a_{11} = a'_{11}a''_{11} + a'_{12}a''_{21}$$

$$a_{12} = a'_{11}a''_{12} + a'_{12}a''_{22}$$

$$a_{21} = a'_{21}a''_{11} + a'_{22}a''_{21}$$

$$a_{22} = a'_{21}a''_{12} + a'_{22}a''_{22}$$

Reference book

- Electric Circuits, James W. Nilsson and Susan A. Riedel, Prentice Hall